HW10 Solution

1 Problem 7.2.4

Soln: Only show $f(x) = x^4$ as one example.

LHS $= \int_0^1 x^4 dx = 1/5$.

RHS $= (1/90)[32(1/4)^4 + 12(1/2)^4 + 32(3/4)^3 + 7] = 1/5$.

2 Problem 7.2.8

Soln:

$f(x) = e^x$: LHS $= \int_0^1 e^x dx = e - 1$, RHS $= A_0 + A_1 e$.

$f(x) = \cos(x\pi/2)$: LHS $= \int_0^1 \cos(x\pi/2)dx = 2/\pi$, RHS $= A_0$.

Solving $A_0 = 2/\pi, A_1 = e^{-1}(e - 1 - A_0) = 1 - 1/e - 2/(\pi e)$.

3 Problem 7.2.11

Soln:

Let $p_1(x) = f(x_1) + f[x_1, x_2](x - x_1)$. Then

$\int_{x_0}^{x_3} p_1(x)dx = f(x_1)(x_3 - x_0) + f[x_1, x_2](x - x_1)^2/2|_{x_0}^{x_3} = [(x_3 - x_0) + (1/2)(x_0 - x_1)^2/(x_2 - x_1)]f(x_1) + [(1/2)(x_3 - x_1)^2/(x_2 - x_1)]f(x_2)$.

4 Problem 7.2.19

Soln: $\int_a^b f(x)dx = \sum_{i=1,\text{odd}}^{n-1} \int_{x_{i-1}}^{x_{i+1}} f(x)dx \approx \sum_{i=1,\text{odd}}^{n-1} (x_{i+1} - x_{i-1})f(x_i) = (2h) \sum_{k=1}^{n/2} f(x_{2k-1})$ since n is even.

5 Problem 7.2.21

Soln:

Bt Theorem 6.1.2, $\int_0^1 f(x)dx = \int_0^1 p_2(x)dx + (1/6)\int_0^1 f'''(\xi)(x - x_0)(x - x_1)(x - x_2)dx$.

Error term: $\leq (M/6)\int_0^1 (x - x_0)(x - x_1)(x - x_2)dx$ where $|f'''(x)| \leq M$ on [0, 1].

If $x_0 = 0, x_1 = 1/2, x_2 = 1$, $\int_0^1 (x - x_0)(x - x_1)(x - x_2)dx = 1/32$.

If $x_0 = 1/4, x_1 = 1/2, x_2 = 3/4$, $\int_0^1 (x - x_0)(x - x_1)(x - x_2)dx = 5/256$. These nodes leads to a smaller const. in the error term.

6 Problem 7.3.8.a

Soln:

$A_0 = A_1 = 1/3$.

7 Problem 7.3.9

Soln:

$\int_{-1}^{1} f(x)dx \approx c[f(x_0) + f(x_1) + f(x_2)]$. $f(x) = 1$: LHS = 2, RHS = $3c \Rightarrow c = 2/3$.

$f(x) = x$: LHS = 0, RHS = $(2/3)[x_0 + x_1 + x_2]$.

$f(x) = x^2$: LHS = 2/3, RHS = $(2/3)[x_0^2 + x_1^2 + x_2^2]$.

There are many possibilities. One example, $x_1 = 0, x_0 = -x_2 = -1/\sqrt{2}$.