HW11

1 Problem 7.1.6

Show that the following two formulas are both $\mathcal{O}(h^4)$ by establish their error terms:

$f'(x) \approx \frac{1}{12h}[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)]$

$f''(x) \approx \frac{1}{12h^2}[-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)]$

2 Problem 7.1.9

Let $\varphi(h) = \frac{1}{2h}[f(x+h) - f(x-h)]$ be the centered difference approximation to $f'(x)$, and $\psi(h) = 4/3\varphi(h/2) - 1/3\varphi(h)$. Show that in Richardson extrapolation, $D(2,2) = 16/15\psi(h/2) - 1/15\psi(h)$.

3 Problem 8.1.3

Compute $x(0.1)$ by solving the differential equation

$$\begin{aligned} x' &= -tx^2 \\ x(0) &= 2 \end{aligned}$$

with one step of the Taylor-series method of order 2. (Hint: You will need to derive the difference equation.)

4 Problem 8.4.4

Use the method of undetermined coefficients to derive the fourth-order Adams-Bashforth formula

$x_{n+1} = x_n + h/24[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$.

(Hint: The basis of the interpolation polynomials is $\{1, t, t(t+1), t(t+1)(t+2)\}$. We can follow p. 550 of the textbook.)