Problem 2.1

34 Soln: Let a positive real number $x = q \times 2^m$, with $1/2 \leq x < 1$. $x = (0.a_1a_2...a_{25}a_{26}...)_2 \times 2^m$ is not a machine number.

Nearby machine numbers: $x_- = (0.a_1a_2...a_{24})_2 \times 2^m$, $x_+ = [(0.a_1a_2...a_{24})_2 + 2^{-24}] \times 2^m$.

To show $fl(x) = x/(1+\delta)$ with $|\delta| \leq 2^{-24}$, it is equivalent to show $(1+\delta)fl(x) = x$, or to show $\delta = (x - fl(x))/fl(x)$ with $|\delta| \leq 2^{-24}$.

Case 1: If we round up, then $fl(x) = x_+$. $\Rightarrow |\delta| = |x - fl(x)|/fl(x) = |x - x_+|/x_+ \leq 2^{m-25}/x_+ < 2^{m-25}/x \leq 2^{-24}$ (The analysis is similar to what we have in the textbook).

Case 2: If $fl(x) = x_-$,

then $\Rightarrow |\delta| = |x - fl(x)|/fl(x) = |x - x_-|/x_- = (0.0...0a_{25}a_{26}...)_2/(0..a_1a_2...a_{24})_2) \leq 2^{-25}/(1/2) = 2^{-24}$.