1. Problem 3.1.4

Determine the formula $n \geq \frac{\log(b_0-a_0)-\log\epsilon}{\log 2} - 1$ involving $b_0 - a_0$ and ϵ for the number steps that must be taken in the bisection method to guarantee that $|r - c_n| \leq \epsilon$.

2. Problem 3.1.11

Give an example (or prove that none exists) in which $a_0 < a_1 < a_2 < ...$ for the bisection method.

3. Problem 3.2.6

To compute reciprocals without division, we can solve $x = 1/R$ by finding a zero of the function $f(x) = x^{-1} - R$. Write a short algorithm to find $1/R$ by Newton's method applied to f. For positive R, what starting points are suitable?

4. Problem 3.2.23

Perform one iteration of Newtion's method on the system,

$$\begin{cases} 4x_1^2 - x_2^2 = 0 \\ 4x_1x_2^2 - x_1 = 1 \end{cases}$$

, starting with $(0, 1)$.

5. Problem 3.3.2

In the secant method, prove that if $x_n \to q$ as $n \to \infty$, and if $f'(q) \neq 0$, then q is a zero of f.

6. Problem 3.3.7

For the secant method, why is the form $x_{n+1} = \frac{f(x_n)x_{n-1}-x_nf(x_{n-1})}{f(x_n)-f(x_{n-1})}$ inferior to $x_{n+1} = x_n - f(x_n)[\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}]$ in practice?

7. Given a real number $c > 0$, find an iteration formula for Newton's method to compute n^{th} root $\sqrt[n]{c}$, with $n > 1$. Does it converge? Explain.

8. Consider a problem of finding the root using the secant method, and nine-decimal-digit floating point arithmetic. If x_n is around 100, is it reasonable to use the termination criterion $|f(x_n)| \leq 10^{-8}$?