HW3 Solution

1. Problem 3.1.4

Soln: Since $|r - c_n| \le 2^{-(n+1)}(b_0 - a_0) \le \varepsilon$, and $-(n+1)\log 2 + \log(b_0 - a_0) \le \log\varepsilon \Rightarrow -(n+1) \le [\log\varepsilon - \log(b_0 - a_0)]/\log 2$. Hence $n > [\log(b_0 - a_0) - \log\varepsilon]/(\log 2) - 1$.

2. Problem 3.1.11

Soln: If $a_0 < a_1 < a_2 < ...$, then $b_n = b_0$ for all n. Therefore the root must be b_0. This is impossible, since we assume that $f(a_0)f(b_0) < 0$.

3. Problem 3.2.6

Soln: $f(x) = x^{-1} - R$. $x_{n+1} = x_n - f(x_n)/f'(x_n) = x_n - (x_n^{-1} - R)/(-x_n^{-2}) = x_n(2 - Rx_n)$.

Algorithm (assuming $R > 1$):

$x_0 \leftarrow 0.1$ // 0.1 is init. guess.

for $k = 1, 2, ...$ until convergence, do

$x_{k+1} = x_k(2 - Rx_k)$

end do

4. Problem 3.2.23

Soln: $J = \begin{bmatrix} 8x_1 & -2x_2 \\ 4x_2^2 - 1 & 8x_1x_2. \end{bmatrix}$. So $J(0,1) = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$, and $J^{-1}(0,1) = 1/6 \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$.

Thus $\begin{bmatrix} h_1^{(1)} \\ h_2^{(2)} \end{bmatrix} = (-1/6)\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -1/2 \end{bmatrix}$. So $\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/2 \end{bmatrix}$.

5. Problem 3.3.2

Soln: $\lim_{n\to\infty}(f(x_n) - f(x_{n-1}))/(x_n - x_{n-1}) = f'(q)$ and $\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} x_n = q$. Thus $x_{n+1} = x_n - f(x_n)[x_n - x_{n-1}]/[f(x_n) - f(x_{n-1})] \Rightarrow q = q - f(q)/f'(q)$. Since $f'(q) \neq 0, f(q) = 0$

6. Problem 3.3.7

Soln: As $x_n \to x_{n+1} \approx r$, $f(x_n) \to f(x_{n+1}) \approx f(r)$ resulting in subtracting close numbers. On the other hand, in (3), x_{n+1} is corrected by a small amount.

7. Soln: It is equivalent to solve $x^n - c = 0$. Since $f(x) = x^n - c$, $f'(x) = nx^{n-1}$, $x_{k+1} = x_k - (x_k^n - c)/(nx_k^{n-1}), k = 0, 1, ...$ In $(0, \infty)$, $f'(x) > 0$, and $f''(x) > 0$, it converges.

8.

Soln: It is not reasonable to use $|f(x_n)| \le 10^{-8}$ as the termination criterion. When $x_n - x_{n-1}$

is subtracted, with nine-decimal-digit arithmetic, the result could be accurate to $\sim 10^{-6}$.