

HW 4

1. Problem 3.4.4

Show that these functions are contractive on the indicated intervals. Determine the best values of λ in $|F(x) - F(y)| \leq \lambda|x - y|$.

- $(1 + x^2)^{-1}$ on an arbitrary interval
- $\frac{1}{2}x$ on $1 \leq x \leq 5$
- $\tan^{-1}x$ on an arbitrary closed interval excluding 0
- $|x|^{3/2}$ on $|x| \leq 1/3$

2. Problem 3.4.6

Consider an iteration function of the form $F(x) = x + f(x)g(x)$, where $f(r) = 0$ and $f^{(1)}(r) \neq 0$. Find the precise conditions on the function g so that the method of functional iteration will converge cubically to r if started near r .

3. Problem 3.4.20

Show that the following functions are contractive on the given domains, yet they have no fixed point on these domains. Why does this not contradict the Contractive Mapping Theorem?

- $F(x) = 3 - x^2$ on $[-1/4, 1/4]$
 - $F(x) = -x/2$ on $[-2, -1] \cup [1, 2]$
4. Assume that fixed point iteration method satisfies $e_{n+1} = k^\alpha e_n$ for some k , $|k| < 1$ and $\alpha > 0$, in which $e_n = x_n - r$, r is the fixed point. Find an expression for the number of iteration N required to reduce the initial error $e_0 = x_0 - r$ by a factor of 10^{-m} , ($m > 0$).

5. Problem 3.5.1

Use Horner's algorithm to find $p(4)$, where $p(z) = 3z^5 - 7z^4 - 5z^3 + z^2 - 8z + 2$.