HW4 Solution

1. Problem 3.4.4

Soln: (a) $F'(x) = -2x(1+x^2)^{-2}$. $\max|F'(x)|$ is obtained when $x = 1/\sqrt{3} \approx 0.65$. Thus $F(x)$ is a contractive mapping. $\lambda = 0.65$

(b) $|F(x) - F(y)| = (1/2)|(y-x)/(xy)| < |y-x|/2$, since x and y are between 1 and 5. $\lambda = 1/2$ from above.

(c) By Mean value theorem, $|F(x) - F(y)| = |F'(\xi)||x-y| = |x-y|/(1+\xi^2)$. Given an interval $[a,b]$, such that $x, y \in [a,b]$, then $\xi \in [a,b]$. So $1/(1+\xi^2) \leq 1/(1+a^2)$. Therefore $\lambda = 1/(1+a^2)$.

(d) By Mean value theorem, $|F(x) - F(y)| = |F'(\xi)||x-y| = 3/2|\xi^{1/2}||x-y|$. Since $x, y \in [-1/3, 1/3]$, so $\xi \in [-1/3, 1/3]$. $\lambda = 3/2|\xi^{1/2}| \leq 3/2\sqrt{1/3} \approx 0.866$.

2. Problem 3.4.6

Soln: We need to have $F^{(1)}(r) = 0, F^{(2)}(r) = 0, F^{(3)}(r) \neq 0$. From $F^{(1)}(r) = 0, \Rightarrow g(r) = -1/f^{(1)}(r)$. From $F^{(2)}(r) = 0, \Rightarrow g^{(1)} = f^{(2)}(r)/[2(f^{(1)}(r))^2]$.

3. Problem 3.4.20

Soln: (a) $|F(x) - F(y)| = |x^2 - y^2| = |x-y||x+y| \leq (1/2)|x-y|$, since $|x+y| \leq |x| + |y|$, and $|x| < 1/4$ and $|y| < 1/4$. F is a contracting mapping, but $F(0) = 3$, which means F does not map the interval $[-1/4, 1/4]$ into $[-1/4, 1/4]$.

(b) $|F(x) - F(y)| = |x-y|/2$. F is a contraction. Since $F(-1) = 1/2$, F does not map the set $[-2,-1] \cup [1,2]$ into $[-2,-1] \cup [1,2]$.

4.

Soln: $e_{n+1} = k^\alpha e_n \Rightarrow en = (k^\alpha)^n e_0$. If we need to have $e_n < 10^{-m} e_0$, $\Rightarrow (k^\alpha)^n < 10^{-m}$. $\Rightarrow n\alpha \log_{10} k < -m$. Since $|k| < 1$, $\Rightarrow n\alpha > -m/\log_{10} k$.

5. Problem 3.5.1

Soln: $p(4) = 946$.

	3	-7	-5	1	-8	2
4		12	20	60	244	944
	3	5	15	61	236	946