HW5 Solution

1. Problem 4.2.1

Soln: Since $UU^{-1} = I$, column i of U^{-1} corresponds to the system

$$\begin{pmatrix} u_{11} & u_{12} & ... & ... & u_{1n} \\ & u_{22} & ... & ... & u_{2n} \\ & & u_{ii} & ... & u_{in} \\ & & & & u_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_i \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$ Solving this system using back substitution, $x_n = x_{n-1} = ... = x_{i+1} = 0$. so U^{-1} is upper triangular.

2. Problem 4.2.13

Soln: Suppose $\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}$. Then $u_{11} = 0, u_{12} = a, l_{21}u_{11} = 0, l_{21}u_{12} + u_{22} = b$. If l_{21} is chosen to be any number, then $u_{11} = 0$, and $u_{22} = b - l_{21}u_{12}$.

3. Problem 4.2.30

Soln: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & -3 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 26/3 \end{pmatrix}$

4. Problem 4.3.1

Soln: $LU = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 3/2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & -4 \\ 0 & 4 & -8 \\ 0 & 0 & 2 \end{pmatrix}$, $x = (5/4, -3/4, -1/2)^T$.