HW6

1. Problem 4.3.9

Show how Gaussian elimination with scaled row pivoting works on this example (forward phase only): $A = \begin{pmatrix} 2 & -2 & -4 \\ 1 & 1 & -1 \\ 3 & 7 & 5 \end{pmatrix}$. Display the scale array (s_1, s_2, s_3) and the final permutation array (p_1, p_2, p_3). Show the final A-matrix, with multipliers stored in the correct locations.

2. Problem 4.3.16

Show how Gaussian elimination with scaled row pivoting works on this example: $A = \begin{pmatrix} -9 & 1 & 17 \\ 3 & 2 & -1 \\ 6 & 8 & 1 \end{pmatrix}$. Display the scale array (s_1, s_2, s_3) and the final permutation array (p_1, p_2, p_3). Show the final A-matrix, with multipliers stored in the correct locations. Determine P, L, and U, and verify that $PA = LU$.

3. Problem 4.3.25 (a,b)

a. Show that if we apply Gaussian elimination without pivoting to a symmetric matrix A, then $l_{i,1} = a_{1,i}/a_{11}$.

b. From this, show that if the first row and column of $A^{(2)}$ are removed, the remaining $(n-1) \times (n-1)$ matrix is symmetric.

4. Problem 4.3.28

Determine $\det(A)$, where $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ without computing the determinant by expansion by minors. (Hint: Consider LU factorization).

5. Problem 4.3.41

In a diagonally dominant matrix A, define **the excess in row** i by the equation $e_i = |a_{ii}| - \sum_{j=1, j\neq i}^{n} |a_{ij}|$. Show that in the proof of Theorem 5 the following is true:

$|a_{ii} - a_{i1}a_{1i}/a_{11}| \geq \sum_{j=2, j\neq i}^{n} |a_{ij} - a_{i1}a_{1j}/a_{11}| + e_i$ Thus, the excess in row i is not diminished in Gaussian elimination.