HW6 Soln

1. Problem 4.3.9

$\begin{pmatrix} (2) & -4 & -2 \\ 1 & 1 & -1 \\ (3) & (-1) & 6 \end{pmatrix}$ with multipliers stored. $s = (4, 1, 7), p = (2, 1, 3)$.

2. Problem 4.3.16

$\begin{pmatrix} (3) & (7/4) & 35/4 \\ 3 & 2 & -1 \\ (2) & 4 & 3 \end{pmatrix}$ with multipliers stored. $s = (17, 3, 8), p = (2, 3, 1)$.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 7/4 & 1 \end{pmatrix}, U = \begin{pmatrix} 3 & 2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & 35/4 \end{pmatrix}$$

3. Problem 4.3.25(a,b)

(a). $l_{i1} = a_{i1}/a_{11}$. Since $A = A^T$, $a_{ij} = a_{ji}$, so $l_{j1} = a_{1j}/a_{11}$.

(b). $a_{ij}^{(2)} = a_{ij}^{(1)} - l_{i1}a_{1j}^{(1)}$; $a_{ji}^{(2)} = a_{ji}^{(1)} - l_{j1}a_{1i}^{(1)}$ for $i, j = 2, ..., n$. $l_{i1}a_{1j}^{(1)} = [a_{1i}^{(1)}/a_{11}^{(1)}]/a_{1j}^{(1)}$, and $l_{j1}a_{1i}^{(1)} = [a_{1j}^{(1)}/a_{11}^{(1)}]/a_{1i}^{(1)}$.

4. Problem 4.3.28

$$A = LU.\ L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 2/7 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 7/2 & 1 \\ 0 & 0 & 12/7 \end{pmatrix}.\ \det(A) = 12.$$

5. 4.3.41

$e_i = |a_{ii}| - \sum_{j=1,j\neq i}^n |a_{ij}|, e_1 = |a_{11}| - \sum_{j=2}^n |a_{1j}|, |a_{11}| = \sum_{j=2}^n |a_{1j}| + e_1, 1 = \sum_{j=2}^n |a_{1j}/a_{11}| + e_1/|a_{11}|, |a_{i1}| = \sum_{j=2}^n |a_{i1}a_{1j}/a_{11}| + |a_{i1}|e_1/|a_{11}|, e_i = |a_{ii}| - \sum_{j=1,j\neq i}^n |a_{ij}|, |a_{ii}| - \sum_{j=2,j\neq i}^n |a_{ij}| = |a_{i1}| + e_i = \sum_{j=2}^n |a_{i1}a_{1j}/a_{11}| + |a_{i1}|e_1/|a_{11}| + e_i$. So $|a_{ii} - a_{i1}a_{1i}/a_{11}| \geq |a_{ii}| - |a_{i1}a_{1i}/a_{11}| = \sum_{j=2,j\neq i}^n [|a_{ij}| + |a_{i1}a_{1j}/a_{11}|] - \sum_{j=2,j\neq i}^n |a_{i1}a_{1j}/a_{11}| + |a_{i1}| + e_i \geq \sum_{j=2,j\neq i}^n [|a_{ij}| + |a_{i1}a_{1j}/a_{11}|] + e_i$,since $1 \geq \sum_{j=2}^n |a_{1j}/a_{11}|$, $a_{i1} \geq \sum_{j=2}^n |a_{i1}a_{1j}/a_{11}|$.