HW7

1. Problem 4.4.3

Show that $||x||_1 \leq n||x||_\infty$ and $||x||_2 \leq \sqrt{n}||x||_\infty$ for all $x \in R^n$.

2. Problem 4.4.11

Show that for the vector norm $||x||_1$ defined in Eq. (5), the subordinate matrix norm is $||A||_1 = max_{1\leq i\leq n} \sum_{i=1}^{n} |a_{ij}|$.

3. Problem 4.4.12

Using $||A||_1$ matrix norm, compute the condition number of the matrix $\begin{pmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{pmatrix}$.

4. Problem 4.4.41

Using problems 4.4.2-3 (p193), prove that $n^{-1}||A||_2 \leq n^{-1/2}||A||_\infty \leq ||A||_2 \leq n^{1/2}||A||_1 \leq n||A||_2$.

5. 4.4.41

In solving $Ax = b$ with matrix $\begin{pmatrix} 1 & 2 \\ 1 & 2.01 \end{pmatrix}$, use $||A||_\infty$ to show how slight changes in b will affect the solution in x. Use $b = (4,4)^T$ and $\hat{b} = (3,5)^T$.

6. Problem 4.6.2

Prove that if A is unit row diagonally dominant, then Richardson iteration is successful. (Note: The Richardson iteration formula is Eq. 12).

7. The iteration formula is $x^{(k+1)} = Gx^{(k)} + d$. If $||G|| < 1$, show $||x^{(k)} - x|| \leq \frac{||G||}{1-||G||}||x^{(k)} - x^{(k-1)}||$, where x is the accurate solution.

Hint: The following inequality is also true: $||x^{(k)} - x|| \leq \frac{||G||^k}{1-||G||}||x^{(1)} - x^{(0)}||$. To show this, assume $m > k$, $\Rightarrow x^{(k)} - x^{(m)} = \sum_{i=k}^{m-1}(x^{(i)} - x^{(i+1)})$,

$||x^{(k)} - x^{(m)}|| \leq \sum_{i=k}^{m-1} ||x^{(i)} - x^{(i+1)}|| \leq \sum_{i=k}^{m-1} ||G||^i||x^{(0)} - x^{(1)}|| = ||G||^k \frac{1-||G||^{m-k}}{1-||G||}||x^{(0)} - x^{(1)}||$.

Let $m \to \infty$. $||x^{(k)} - x^{(m)}|| \leq \frac{||G||^k}{1-||G||}||x^{(1)} - x^{(0)}||$.