HW8

1. Problem 6.1.9

Prove that if \( g \) interpolates the function \( f \) at \( x_0, x_1, ..., x_{n-1} \) and if \( h \) interpolates \( f \) at \( x_1, x_2, ..., x_n \), then the function \( g(x) + \frac{x_0-x}{x_n-x_0}[g(x) - h(x)] \) interpolates \( f \) at \( x_0, x_1, ..., x_{n-1}, x_n \).

2. Problem 6.1.10

Prove that the coefficient of \( x^n \) in the Lagrange form of the interpolation polynomial \( p_n \) is \( \sum_{i=0}^{n} y_i \prod_{j=0, j \neq i}^{n} (x_i - x_j)^{-1} \).

3. Problem 6.1.23

Consider the data \( x \) \(-\sqrt{3}/5\) \( 0 \) \( \sqrt{3}/5 \). What is the Newton form of the interpolation polynomial and the Lagrange form of the interpolation polynomial for these data?

4. Given the point values of the function \( y = 2^x \)

\[
\begin{array}{c|cccc}
 x & 0 & 1 & 2 \\
 2^x & 1 & 2 \\
\end{array}
\]

what is the Lagrange form of the interpolation polynomial. Use this interpolation polynomial to compute \( 2^{0.3} \) approximately, and estimate the absolute error.

5. Problem 6.2.3

Let \( f \in C^n[a,b] \). Prove that if \( x_0 \in (a,b) \) and if \( x_1, x_2, ..., x_n \) all converge to \( x_0 \), then \( f[x_0, x_1, ..., x_n] \) will converge to \( f^{(n)}(x_0)/n! \).

6. Problem 6.2.6

Prove that the divided differences are linear maps on functions. That is, \( (\alpha f + \beta g)[x_0, x_1, ..., x_n] = \alpha f[x_0, x_1, ..., x_n] + \beta g[x_0, x_1, ..., x_n] \). (Hint: proof by induction).