HW8

1. Problem 6.1.9

Prove that if g interpolates the function f at $x_0, x_1, ..., x_{n-1}$ and if h interpolates f at $x_1, x_2, ..., x_n$,then the function $g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$ interpolates f at $x_0, x_1, ..., x_{n-1}, x_n$.

2. Problem 6.1.10

Prove that the coefficient of x^n in the Lagrange form of the interpolation polynomial p_n is $\sum_{i=0}^{n} y_i \prod_{j=0,j\neq i}^{n} (x_i - x_j)^{-1}$.

3. Problem 6.1.23

Consider the data

x	-$\sqrt{3/5}$	0	$\sqrt{3/5}$
$f(x)$	$f(-\sqrt{3/5})$	$f(0)$	$f(\sqrt{3/5})$

. What is the Newton form of the interpolation polynomial and the Lagrange form of the interpolation polynomial for these data?

4. Given the point values of the function $y = 2^x$

x	0	1
2^x	1	2

, what is the Lagrange form of the interpolation polynomial. Use this interpolation polynomial to compute $2^{0.3}$ approximately, and estimate the abslute error.

5. Problem 6.2.3

Let $f \in C^n[a, b]$. Prove that if $x_o \in (a, b)$ and if $x_1, x_2, ..., x_n$ all converge to x_0, then $f[x_0, x_1, ..., x_n]$ will converge to $f^{(n)}(x_0)/n!$.

6. Problem 6.2.6

Prove that the divided differences are linear maps on functions. That is, $(\alpha f + \beta g)[x_0, x_1, ..., x_n] = \alpha f[x_0, x_1, ..., x_n] + \beta g[x_0, x_1, ..., x_n]$. (Hint: proof by induction).