HW8 Solution

1. Problem 6.1.9

Soln: Let $k(x) = g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(h)]$. Then we will have $k(x_i) = f(x_i)$.

2. Problem 6.1.10

Soln: $p(x) = \sum_{i=0}^{n} y_i \prod_{j=0,j\neq i}^{n} \frac{(x-x_j)}{(x_i-x_j)}$. Coefficient of x^n is $\sum_{i=0}^{n} y_i \prod_{j=0,j\neq i}^{n} \frac{1}{(x_i-x_j)}$

3. Problem 6.1.23

Soln: Newton form: $p(x) = c_0 + c_1(x + \sqrt{3/5}) + c_2(x + \sqrt{3/5})x$ where $c_0 = f(-\sqrt{3/5}), c_1 = [f(0) - f(-\sqrt{3/5})]/\sqrt{3/5}, c_2 = [f(\sqrt{3/5} - 2f(0) + f(-\sqrt{3/5}))]/[6/5]$.

Lagrange form: $p(x) = f(-\sqrt{3/5})l_0(x) + f(0)l_1(x) + f(\sqrt{3/5})l_2(x)$ where $l_0(x) = (5/6)x(x - \sqrt{3/5}), l_1(x) = -(5/3)(x + \sqrt{3/5})(x - \sqrt{3/5}), l_2(x) = (5/6)(x + \sqrt{3/5})x$.

4.

Soln: $p(x) = x + 1$, $p(0.3) = 1.3$. $e = 2^{0.3} - 1.3 \simeq 0.069$.

5. Problem 6.2.3

Proof: Since $f^{(n)}(x)$ is continuous in a neighborhood of x_0, so $f[x_0, ..., x_0] = (1/n!)f^{(n)}(x_0)$ by Theorem 6.2.4.

6. Problem 6.2.6

Proof: By induction. Check it is true for $n = 1$, trivial. Suppose it is true for 2, 3, ..., n. Consider $(\alpha f + \beta g)[x_0, ..., x_{n+1}] = \{(\alpha f + \beta g)[x_1, ..., x_{n+1}] - (\alpha f + \beta g)[x_0, ..., x_n]\}/(x_{n+1} - x_0) = \{\alpha f[x_1, ..., x_{n+1}] - \alpha f[x_0, ..., x_n] + \beta g[x_1, ..., x_{n+1}] - \beta g[x_0, ..., x_n]\}/(x_{n+1} - x_0) = \alpha f[x_0, ..., x_{n+1}] + \beta g[x_0, ..., x_{n+1}]$.