HW9 Solution

1.

Proof: By Theorem 6.1.2. Since $f^{(n+1)} = 0$, the pointwise interpolation error $\equiv 0$. Therefore $p \equiv f$.

2. Problem 6.3.1

Soln: $p(x) = 2 - 9x + 3x^2 + 7x^2(x-1) + 5x^2(x-1)^2$.

3.

Soln:

Define the function $g(t) = f(t) - P_{m+n+1}(t) - \frac{f(x)-P_{m+n+1}(x)}{\prod_{i=0}^{n}(x-x_i)\prod_{k=0}^{m}(x-x_{i_k})}\prod_{i=0}^{n}(t-x_i)\prod_{k=0}^{m}(t-x_{i_k})$. $x, x_0, x_1, ..., x_n$ are zeros of $g(t)$.

Using Rolle's theorem, $g'(t)$ has at least $n+1$ zeros among $x, x_0, x_1, ..., x_n$. Since x_{i_k} are also zeros of $g^{'}(t)$, there are at least $m+n+2$ zeros. Use Rolle's theorem repeatly, $g^{(m+n+2)}$ has at least one zero ξ.

Let $t = \xi$, plug into $g(t) = ...$, and solve for $f(x) - P_{m+n+1}(x)$. $\Rightarrow f(x) - P_{m+n+1}(x) = \frac{f^{(m+n+2)}(\xi)}{(m+n+2)!}\prod_{i=0}^{n}(x-x_i)\prod_{k=0}^{m}(x-x_{i_k})$.

4. Problem 6.8.15

Proof: Let $u_1, u_2, ..., u_n$ be orthongonal and $u_k \neq 0$ for each k. Suppose that $\sum_j c_j u_j = 0$. By taking inner product with u_k we get $c_k < u_k, u_k >= 0$. Here $c_k = 0$.

5. Problem 5.3.5

Proof:

a. $P(x+y) = \sum_{i=1}^{n} < x+y, u_i > u_i = \sum_{i=1}^{n} < x, u_i > u_i + \sum_{i=1}^{n} < y, u_i > u_i = P(x) + P(y)$.

b. $P(P(x)) = \sum_{i=1}^{n} < \sum_{j=1}^{n} < x, u_j > u_j, u_i > u_i = \sum_{i=1}^{n} < x, u_i > u_i = Px$.

c. if $x \in U$, $x = \sum_{i=1}^{n} < x, u_i > u_i = Px$.

d. $||Px||_2^2 =< Px, Px >= \sum_{i=1}^{n} | < x, u_i > |^2$. $||x||_2^2 = ||x - Px||_x^2 + ||Px||_2^2 \geq ||Px||_2^2 = \sum_{i=1}^{n} | < x, u_i > |^2$.