Math 20580 practice exam 1, Spring 2011

1. Find all solutions to $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$, (b) $\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$, (c) $t \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$, (d) no solution, (e) $\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

2. The solution set of the linear system $\begin{bmatrix} -2 & 8 & 1 \\ 3 & 5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is

(a) a straight line, (b) a point, (c) a plane,
(d) no solution, (e) has exactly two solutions

3. The coefficient matrix of a linear transformation $T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ -x_1 + 3x_2 - 7x_3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 2 & -3 \\ -1 & 3 & -7 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ -3 & 7 \end{bmatrix}$, (c) $\begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ -x_1 + 3x_2 - 7x_3 \end{bmatrix}$,

(d) 0, (e) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

4. The set of row-vectors $\{[1 \quad 2 \quad 3], [0 \quad 0 \quad 0], [2 \quad 1 \quad 4]\}$ is
(a) linearly dependent, (b) linearly independent, (c) spans the whole $\mathbf{R}^3$,
(d) forms a basis of $\mathbf{R}^3$, (e) orthogonal to $[1 \quad 0 \quad 0]$

5. Let $T\mathbf{v} = A\mathbf{v}$ be the flection through x_2-axis and $T : \mathbf{R}^2 \to \mathbf{R}^2$. Then the matrix A is

(a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, (b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, (c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$,

(d) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, (e) does not exist

6. The map $T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 + 2x_2 \\ 2x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$ has the following property

(a) the map T is one-to-one, (b) the map T is onto,

(c) $Nul(A)$ has dimension 1, (d) $Col(A)$ has dimension 1,
(e) $A\mathbf{x} = 0$ has no solution

7. Let $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$. Then A^3 is equal to

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, (b) $\begin{bmatrix} 0 & 8 \\ 0 & 0 \end{bmatrix}$, (c) $\begin{bmatrix} 0 & 0 \\ 8 & 0 \end{bmatrix}$,
(d) $\begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix}$, (e) does not make sense

8. Let $H = \{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid x_1^2 - x_2^2 = 0\}$ be a subset of $\mathbf{R}^2$. Then H is

(a) not a linear subspace, (b) a linear subspace, (c) a circle,
(d) a plane, (e) a paraboloid

9. Let $A = \begin{bmatrix} 1 & 1 & 9 \\ 2 & 3 & 7 \end{bmatrix}$. The dimension of the null space $Nul(A)$ is equal to

(a) 1, (b) 4, (c) 2, (d) 3, (e) 0

10. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ Then the verse matrix A^{-1} of A is equal to

(a) does not exist, (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,
(d) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, (e) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

11. Suppose that the system $\begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ h \end{bmatrix}$ is solvable. Then $h =$

(a) -4, (b) 4, (c) -2, (d) 2, (e) 0

12. Let $A = \begin{bmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix}$. The rank of A is

(a) 2, (b) 3, (c) 1, (d) 4, (e) 0

Partial credit problems:

13. Find the inverse of $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 2 \\ 7 & 2 & 5 \end{bmatrix}$.

14. Let $\mathbf{B} = \{ \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} \}$ and $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Find the $\mathbf{B}$-coordinates $[\mathbf{x}]_{\mathbf{B}}$ of $\mathbf{x}$.

15. Let $A = \begin{bmatrix} -3 & 6 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 8 \end{bmatrix}$. Find a basis of the column space $Col(A)$ of A.