Math 20580 practice exam 2, Spring 2011

1. Let S be the parallelogram determined by the vectors $\mathbf{b}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and let $A = \begin{pmatrix} 4 & 7 \\ 0 & 2 \end{pmatrix}$. Compute the area of the images of S under the mapping $\mathbf{v} \longmapsto A\mathbf{v}$.

(a) 8 (b) -8 (c) 7 (d) 4 (e) 2

2. For what value(s) of h will $\mathbf{y}$ be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, if $\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 3 \\ 5 \\ h \end{pmatrix}$.

(a) 8 (b) 5 (c) 4 (d) 1 (e) 2

3. Find a matrix A such that $W = Col(A)$ where $W = \{ \begin{pmatrix} 7a+b \\ 6a-5b \\ 7b \end{pmatrix} \}$ and $\{a, b\}$ range over all real numbers.

(a) $\begin{pmatrix} 7 & 1 \\ 6 & -5 \\ 0 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} 7 & 6 & 0 \\ 1 & -5 & 7 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (e) $\begin{pmatrix} 7 & 1 \\ 6 & -5 \\ 7 & 0 \end{pmatrix}$

4. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$. Then

(a) rank(A) = 2 (b) Col(A) = R^3 (c) $\det A = 1$

(d) A^{-1} exists (e) $rank(A) = 3$

5. Let $\mathbf{b}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find $[\mathbf{x}]_{\mathbf{B}}$.

(a) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (e) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

6. Let $A = \begin{pmatrix} 7 & 2 & 3 \\ 1 & 5 & 3 \\ 8 & 7 & 6 \end{pmatrix}$. The rank of A is

(a) 2 (b) 3 (c) 4 (d) 1 (e) 0

7. Let $\mathbf{b}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{b}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\mathbf{c}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{c}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$. If $P = [[\mathbf{b}_1]_{\mathbf{C}}, [\mathbf{b}_2]_{\mathbf{C}}]$ is the change-of-coordinates matrix from $\mathbf{B}$ to $\mathbf{C}$, then find P.

(a) $\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 2 & -3 \\ 3 & 5 \end{pmatrix}$

8. Let $\{\lambda_1, \lambda_2\}$ be two eigenvalues of $A = \begin{pmatrix} 3 & 5 \\ 5 & 7 \end{pmatrix}$. Then the sum of two eigenvalues, $(\lambda_1 + \lambda_2)$, is equal to

(a) 10 (b) -4 (c) 3 (d) 7 (e) 5

9. Let $A = \begin{pmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{pmatrix}$. The eigenvalues of A are

(a) $1, 2, 2$ (b) $1, -2, -2$ (c) $1, 2, 3$ (d) $2, 3, -2$ (e) $0, 1, 2$

10. Let $A = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}$. The complex eigenvalues of A are

(a) $1 \pm i$ (b) $3 \pm i$ (c) $2 \pm i$ (d) $0, 1$ (e) $\pm i$

11. The matrix $A = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}$ has a complex eigenvector:

(a) $\begin{pmatrix} 5 \\ 2+i \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 3i \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 5i \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

12. Let $\mathbf{P}_2 = \{a_0 + a_1 t + a_2 t^2\}$ where $\{a_0, a_1, a_2\}$ range over all real numbers, and let $T : \mathbf{P}_2 \to \mathbf{P}_2$ be a linear transformation defined by

$$T(a_0 + a_1 t + a_2 t^2) = a_0 + a_1 t + a_0 t^2.$$

If $T(p(t)) = \lambda p(t)$ for some non-zero polynomial $p(t)$ in $\mathbf{P}_2$ and some real number λ, then $p(t)$ is called an eigenvector of T corresponding to λ. The linear transformation T has an eigenvector:

(a) $1 + t + t^2$ (b) $1 + t$ (c) $t + t^2$ (d) 100 (e) 0

Partial credit problems:

13. Let $A = \begin{pmatrix} 1 & -4 & 2 \\ -1 & 7 & 0 \\ -2 & 8 & -9 \end{pmatrix}$. Use Cramer's rule to solve $A\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

14. Let $P_2 = \{a_0 + a_1 t + a_2 t^2\}$ where $\{a_0, a_1, a_2\}$ range over all real numbers, and let $T : P_2 \to P_0$ be a linear transformation given by $T(a_0 + a_1 t + a_2 t^2) = \int_0^1 (a_0 + a_1 t + a_2 t^2) dt$. Suppose that $\mathbf{B} = \{1, t, t^2\}$ is a basis of P_2 and $\mathbf{C} = \{1\}$ is a basis of P_0.
(1) Find a matrix A such that $[T(\mathbf{v})]_{\mathbf{C}} = A[\mathbf{v}]_{\mathbf{B}}$;
(2) Find $Nul(A)$ and $Col(A)$.

15. Let $A = \frac{1}{4}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$. Find $\lim_{k \to \infty} A^k$. (Hint: Find the Diagonalization D of A and use the formula $A^k = PD^kP^{-1}$).