Math 20580 practice exam 3, Spring 2011

1. Let $A = \frac{1}{7}\begin{pmatrix} 2 & 6 & 3 \\ 3 & 2 & -6 \\ 6 & -3 & 2 \end{pmatrix}$. Find A^{-1}.

(a) A^T (b) $7\begin{pmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{pmatrix}$ (c) A^2

(d) A^3 (e) $\begin{pmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{pmatrix}$

2. Let $W = \text{Span}\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\}$. Find the orthogonal complement of W in $\mathbb{R}^3$.

(a) $\text{Span}\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}\}$ (b) $\text{Span}\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}\}$

(c) $\text{Span}\{\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}\}$ (d) $\text{Span}\{\begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}\}$

(e) $\text{Span}\{\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}\}$

3. Solve equation $y' = 9.8 - \frac{y}{5}$ with initial condition $y(0) = 50$.

(a) $49 + e^{-\frac{t}{5}}$ (b) 50 (c) $1 + 49e^{-\frac{t}{5}}$ (d) 49 (e) 9.8

4. Let $\mathbf{y} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}$, $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ and $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Use the fact that $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal to compute $\text{Proj}_W \mathbf{y}$.

(a) $\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ (d) $\begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}$ (e) $\begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}$

5. Find $\vec{u}_3$ so that the subset $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}, \vec{u}_3\}$ becomes an orthogonal basis of $W =$ $\text{Span}\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\3\\1\\7\end{pmatrix}\}$

(a) $\begin{pmatrix}-3\\1\\-1\\3\end{pmatrix}$ (b) $\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ (c) $\begin{pmatrix}1\\3\\1\\7\end{pmatrix}$ (d) $\begin{pmatrix}4\\2\\2\\4\end{pmatrix}$ (e) $\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$

6. Find an orthonormal basis of the subspace $W = \text{Span}\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$.

(a) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \frac{1}{2}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\}$ (b) $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\}$ (c) $\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$

(d) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\9\\-9\\0\end{pmatrix}\}$ (e) $\{\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\9\\9\\1\end{pmatrix}\}$

7. Let $\mathbf{y} = \begin{pmatrix}6\\7\end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix}2\\4\end{pmatrix}$. Find orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$.

(a) $\begin{pmatrix}4\\8\end{pmatrix}$ (b) $\begin{pmatrix}8\\4\end{pmatrix}$ (c) $\begin{pmatrix}2\\4\end{pmatrix}$ (d) $\begin{pmatrix}0\\0\end{pmatrix}$ (e) $\begin{pmatrix}4\\2\end{pmatrix}$

8. Find a least-square solution of inconsistent system $A\mathbf{x} = \mathbf{b}$ for $A = \begin{pmatrix}0&4\\2&0\\1&1\end{pmatrix}$ and $\mathbf{b} =$ $\begin{pmatrix}2\\0\\11\end{pmatrix}$.

(a) $\begin{pmatrix}2\\1\end{pmatrix}$ (b) $\begin{pmatrix}0\\1\\2\end{pmatrix}$ (c) $\begin{pmatrix}2\\1\\0\end{pmatrix}$ (d) $\begin{pmatrix}1\\2\end{pmatrix}$ (e) $\begin{pmatrix}0\\0\end{pmatrix}$

9. Solve the initial value problem of $ty' + 2y = 4t^2$ with the initial condition $y(1) = 3$.

(a) $t^2 + \frac{2}{t^2}$ (b) $2t^2 + \frac{1}{t^2}$ (c) $t^2 + \frac{1}{t^2}$ (d) $t^2 - \frac{2}{t^2}$ (e) $t^2 - \frac{1}{t^2}$

10. Find all solutions to the separable equation $y' = \dfrac{x^2}{y(1+x^3)}$.

(a) $3y^2 - 2\ln|1+x^3| = c$ (b) $3y^2 - \ln|1+x^3| = c$ (c) $2y^2 - 3\ln|1+x^3| = c$

(d) $y^2 - 2\ln|1+x^3| = c$ (e) 0

11. Find the distance between $\mathbf{y}$ and W, where $\mathbf{y} = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 13 \end{pmatrix}$, $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}$

and $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

(a) 8 (b) 0 (c) 3 (d) 1 (e) 13

12. Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously. Find the rate of return r that must be achieved if the initial investment is to double in 10 years.

(a) $\frac{\ln 2}{10}$ (b) 20% (c) 10% (d) 2 (e) $\frac{\ln 10}{2}$

13. Let $y = \phi(t)$ be the solution of the initial value problem:

$$y'' - 3y' + 2y = 0, \qquad y(0) = 0, \ \ y'(0) = 1.$$

Find $\phi(\ln 2)$.

(a) 2 (b) 1 (c) 0 (d) 3 (e) 4

Partial Credit Problems:

14. Find the solution of $\dfrac{dy}{dt} = \dfrac{1}{2}(1-y)y$ with $y(0) = 4$ and find $\lim_{t\to\infty} y(t)$.

15. Find an integrating factor for the equation $(xy + x + 1)dx + (x^2 + xy)dy = 0$ and then solve the equation.

16. Find the smallest value for $\|A\mathbf{x} - \mathbf{b}\|$, where $\mathbf{x}$ belongs to $\mathbb{R}^2$, $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$. (Hint: First find $\mathbf{x}$ which realizes the smallest value for $\|A\mathbf{x} - \mathbf{b}\|$ by solving a least square problem).