Spanning set theorem (Section 4.3)

Theorem 4.5. Let the set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a set in V. Let $H = \text{Span}\,\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$.

a. If one of the vectors in S, i.e. $\mathbf{v}_k$ is a linear combination of the remaining vectors in *S*, then the set formed from S by removing $\mathbf{v}_k$ still spans H.
b. If $H \neq \{\mathbf{0}\}$, some subset of *S* is a basis for *H*.

Example. Use Theorem 4.5 to find a basis for Col B, where

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \ldots & \boldsymbol{b}_5 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution: Since $\boldsymbol{b}_2 = 4\boldsymbol{b}_1; \boldsymbol{b}_4 = 2\boldsymbol{b}_1 - \boldsymbol{b}_3,$

Basis = $\{\boldsymbol{b}_1, \boldsymbol{b}_3, \boldsymbol{b}_5\}$

Theorem 4.6. The pivot columns of a matrix A form a basis for Col A.