Sufficient condition for a matrix to be diagonalizable

**Theorem 5.6** An \( n \times n \) matrix with \( n \) distinct eigenvalues is diagonalizable.

Example. Determine if the following matrix is diagonalizable.

\[
A = \begin{bmatrix}
5 & -8 & 1 \\
0 & 0 & 7 \\
0 & 0 & -2
\end{bmatrix}
\]

**Solution:** Eigenvalues of \( A \) are 5, 0, -2. Since \( A \) is a 3 \( \times \) 3 matrix with three distinct eigenvalues, \( A \) is diagonalizable.

**Theorem 5.7** Let \( A \) be a \( n \times n \) matrix whose distinct eigenvalues are \( \lambda_1, ..., \lambda_p \).

a. For \( 1 \leq k \leq p \), the dimension of the eigenspace for \( \lambda_k \) is less than or equal to the multiplicity of the eigenvalue \( \lambda_k \).

b. The matrix \( A \) is diagonalizable if and only if the sum of the dimensions of the distinct eigenspaces equals \( n \), and this happens if and only if the dimension of the eigenspace for each \( \lambda_k \) equals the multiplicity of \( \lambda_k \).

c. If \( A \) is diagonalizable and \( B_k \) is a basis for the eigenspace corresponding to \( \lambda_k \) for each \( k \), then the total collection of vectors in the sets \( B_1, ..., B_p \) forms an eigenvector basis for \( \mathbb{R}^n \).