Sections 5.3

Sufficient condition for a matrix to be diagonalizable

Theorem 5.6 An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.

Example. Determine if the following matrix is diagonalizable.

$$\boldsymbol{A} = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}$$

Solution: Eigenvalues of ***A*** are 5, 0, -2. Since ***A*** is 3 × 3 matrix with three distinct eigenvalues, ***A*** is diagonalizable.

Theorem 5.7 Let ***A*** be $n \times n$ matrix whose distinct eigenvalues are $\lambda_1, \ldots, \lambda_p$.

a. For $1 \leq k \leq p$, the dimension of the eigenspace for λ_k is less than or equal to the multiplicity of the eigenvalue λ_k.

b. The matrix ***A*** is diagnolizable if and only if the sum of the dimensions of the distinct eigenspaces equals n, and this happens if and only if the dimension of the eigenspace for each λ_k equals the multiplicity of λ_k.

c. If ***A*** is diagonalizable and B_k is a basis for the eigenspace corresponding to λ_k for each k, then the total collection of vectors in the sets $B_1, \ldots B_p$ forms an eigenvector basis for R^n.