Theorem 2.4.1 (Existence and Uniqueness of solutions of 1\textsuperscript{st} order linear differential equations). For the 1\textsuperscript{st} order differential equation \( \frac{dy}{dt} + p(t)y = g(t) \), if \( p(t) \) and \( g(t) \) are continuous on an open interval \( I: \alpha < t < \beta \) containing the point \( t = t_0 \), \textit{then there exists a unique function} \( y = \phi(t) \) that satisfies the differential equation for each \( t \) in the interval \( I \), and that also satisfies the initial condition \( y(t_0) = y_0 \), where \( y_0 \) is an arbitrary prescribed initial value.

Remark: Let \( \mu(t) = e^{\int_{t_0}^{t} p(s)ds} \), then the solution of the IVP in Thm 2.4.1 is

\[
y = \frac{1}{\mu(t)} \int_{t_0}^{t} \mu(s)g(s)ds + y_0
\]
Theorem 2.4.2 (Existence and Uniqueness of solutions of 1st order nonlinear differential equations). Let functions $f(t,y)$ and $\frac{\partial f(t,y)}{\partial y}$ be continuous in some rectangle $\alpha < t < \beta, \gamma < y < \delta$ containing the point $(t_0,y_0)$. Then in some interval $t_0 - h < t < t_0 + h$ contained in $\alpha < t < \beta$, there is a unique solution $y = \phi(t)$ of the initial value problem $\frac{dy}{dt} = f(t,y), \ y(t_0) = y_0$.

Remark:

1. For Theorem 2.4.2, existence of a solution (but not its uniqueness) can be established on the basis of the continuity of function $f(t,y)$ alone.

2. Graphs of two solutions cannot intersect each other.