

Key concept: Let $y_1(t)$ and $y_2(t)$ are two solutions of $L[y] = y'' + p(t)y' + q(t)y = 0$, $\alpha < t < \beta$. $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions to the differential equation (or $y(t) = c_1y_1(t) + c_2y_2(t)$ is a general solution) if $W(y_1, y_2)(t_0)$ is NOT zero for some point t_0 , where $\alpha < t_0 < \beta$.

Theorem 3.2.5 Consider the differential equation $L[y] = y'' + p(t)y' + q(t)y = 0$, whose coefficients $p(t)$ and $q(t)$ are continuous on an open interval $I: \alpha < t < \beta$. Choose some point t_0 in I .

Let $y_1(t)$ be the solution of $L[y] = 0$ that also satisfies the initial conditions $y(t_0) = 1, y'(t_0) = 0$.

Let $y_2(t)$ be the solution of $L[y] = 0$ that also satisfies the initial conditions $y(t_0) = 0, y'(t_0) = 1$.

Then $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions of $L[y] = 0$.

Theorem 3.2.6 (Abel's Theorem). If $y_1(t)$ and $y_2(t)$ are solutions of the differential equation $L[y] = y'' + p(t)y' + q(t)y = 0$, whose coefficients $p(t)$ and $q(t)$ are continuous on an open interval $I: \alpha < t < \beta$. Then the Wronskian $W(y_1, y_2)(t)$ is given by $W(y_1, y_2)(t) = C \exp[-\int p(t)dt]$, where C is a certain constant that depends on $y_1(t)$ and $y_2(t)$ but NOT on t . Further, $W(y_1, y_2)(t)$ either is zero for all t in I (if $C = 0$) or else is never zero in I (if $C \neq 0$).