# Theorem 4 in Section 1.4

Example 3b. Let A = $\begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -8 \end{bmatrix}$ and **b** = $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Is the equation A**x** = **b** consistent for all $b_1$, $b_2$, $b_3$?

Solution:

$$\begin{bmatrix} 1 & 3 & 4 & b_1 \\ -4 & 2 & -6 & b_2 \\ -3 & -2 & -8 & b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 4 & b_1 \\ 0 & 14 & 10 & b_2+4b_1 \\ 0 & 7 & 4 & b_3+3b_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 4 & b_1 \\ 0 & 14 & 10 & b_2+4b_1 \\ 0 & 0 & -1 & b_3+b_1-\dfrac{b_2}{2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 2b_1+\dfrac{12}{14}b_3-\dfrac{16}{14}b_2 \\ 0 & 1 & 0 & \dfrac{1}{14}(10b_3+14b_1-4b_2) \\ 0 & 0 & 1 & -b_3-b_1+\dfrac{b_2}{2} \end{bmatrix}$$

The equation is consistent for all $b_1$, $b_2$, $b_3$.

**Theorem 4.** If A is a $m \times n$ matrix, with columns $\mathbf{a}_1, \ldots, \mathbf{a}_n$, the following statements are equivalent.

(a) For each **b** in $R^m$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has a solution

(b) Each **b** in $R^m$ is a linear combination of the columns of A.

(c) The columns of A span $R^m$ (every vector **b** in $R^m$ is a linear combinations of the columns of A).

(d) A has a pivot position in every row.

Note: Compare Theorem 4 with Example 3 and 3b.