# Subspace of $R^n$ (Section 2.8)

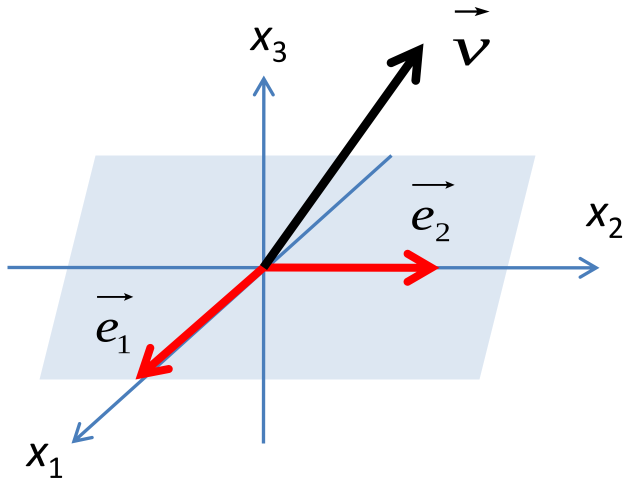
**Definition:** A **subspace** of $R^n$ is any set $H$ in $R^n$ that satisfies:

a. The zero vector is in *H*.

b. For each $\vec{u}$ and $\vec{v}$ in *H*, the sum $\vec{u}+\vec{v}$ is in *H*.

c. For each $\vec{u}$ and each scalar *c*, $c\vec{u}$ is in *H*.

*In short:* all linear combinations $c\vec{\mathbf{u}}+d\vec{\mathbf{v}}$ are in *H*.




**Example.** Let $H$ = Span{ $\vec{e_1}$ , $\vec{e_2}$ }. *H* is a subspace in $R^3$ . $\vec{v}$ is **NOT** in *H*.
Here,

$$\vec{e_1}=\begin{bmatrix}1\\0\\0\end{bmatrix}, \vec{e_2}=\begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \vec{v}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

# Dimension and rank (Section 2.9)

**Fact:** A vector in subspace *H* can be represented in **only one way** as a linear combination of basis vectors of *H*.

**Example**. Let {$\mathbf{e}_1$, $\mathbf{e}_2$} be a basis for subspace *H* = Span{$\mathbf{e}_1$, $\mathbf{e}_2$} in $R^3$.

$$\boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \qquad \boldsymbol{b} = 2\boldsymbol{e}_1 + 3\boldsymbol{e}_2$$

**Definition**: Let the set *B* = {$\mathbf{b}_1$, …, $\mathbf{b}_p$} be a basis for subspace *H*. For each ***x*** in *H*, ***x*** = $c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \ldots + c_p\mathbf{b}_p$. The **coordinates** of ***x*** **relative to the basis** *B* are the weights $c_1$, $c_2$, …, $c_p$. The vector in $R^p$

$$[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{c}_1 \\ \vdots \\ \boldsymbol{c}_{\boldsymbol{p}} \end{bmatrix}$$

is called the **coordinate vector of *x*** (***relative to B***) or the *B*-**coordinate vector** of ***x***