Dimension and rank (Section 2.9) cont'd

Theorem 2.15 The Basis Theorem. Let *H* be a p-dimensional subspace of R^n. Any linearly independent set of exactly *p* elements in H is automatically a basis for *H*. Also, any set of p elements of H that spans H is automatically a basis for *H*.

Theorem The Invertible Matrix Theorem (cont'd). Let the A be a n × n matrix. Then the following statements are each equivalent to the statement that A is an invertible matrix.

m. The columns of A form a basis of R^n.
n. Col A = R^n
o. dim (Col A) = n
p. rank A = n
q. Nul A = {**0**} "**0** here is the zero vector".
r. dim (Nul A) = 0

Introduction to determinant (Section 3.1)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \sim \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{11}a_{22} - a_{12}a_{21} & a_{11}a_{23} - a_{13}a_{21} \\ 0 & 0 & a_{11}\Delta \end{bmatrix}$$

$$\Delta = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Since A is invertible, Δ must be **nonzero**.
Δ is the determinant of the 3 × 3 matrix A, denoted by det A.

Submatrix A_{ij} of A: matrix formed by deleting the i^{th} row and j^{th} column of A.

Example.

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \qquad A_{23} = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$$