Answer Key 1

MATH 20580: Introduction to Linear Algebra and Differential Equations
Practice Exam 2 *March 22, 2011*

1.	a	b	c	•	e	7.	a	b	c	d	•
2.	•	b	c	d	e	8.	a	•	c	d	e
3.	a	b	•	d	e	9.	a	b	c	•	e
4.	a	•	c	d	e	10.	a	b	•	d	e
5.	a	b	c	d	•	11.	•	b	c	d	e
6.	a	b	•	d	e	12.	•	b	c	d	e

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1. Let $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & 2 \\ 4 & 5 & 1 \end{bmatrix}$. The rank of A is

(a) 4 (b) 0 (c) 3 (d) 2 (e) 1

2. Let $\mathbb{P}_2 = \{a_0 + a_1 t + a_2 t^2 \mid a_0, a_1, a_2 \text{ in } \mathbb{R}\}$ and let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be a linear transformation defined by

$$T(a_0 + a_1 t + a_2 t^2) = a_1 + 9a_2 t$$

If $T(p(t)) = \lambda p(t)$ for some non-zero polynomial $p(t)$ in $\mathbb{P}_2$ and some real number λ, then $p(t)$ is called an eigenvector of T corresponding to λ. The linear transformation T has an eigenvector:

(a) 100 (b) t^2 (c) $t + 9t^2$ (d) t (e) 0

3. Let $\{\lambda_1, \lambda_2\}$ be two eigenvalues of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$. Then the product of two eigenvalues, $\lambda_1 \lambda_2$, is equal to

(a) 28 (b) 7 (c) -28 (d) 3 (e) 4

4. Let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find $[\mathbf{x}]_{\mathcal{B}}$.

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

5. The matrix $A = \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$ has a complex eigenvector:

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 3i \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ 4i \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ i \end{bmatrix}$

6. Let $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. The eigenvalues of A are

(a) $0, 1, -2$ (b) $0, 1, 2$ (c) $1, -2, -2$ (d) $1, 2, 3$ (e) $1, \pm 2$

7. Let S be the parallelogram determined by the vectors $\mathbf{b}_1 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $b_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and let $A = \begin{bmatrix} -2 & 99 \\ 0 & 1 \end{bmatrix}$. Compute the area of the images of S under the mapping $\mathbf{v} \to A\mathbf{v}$.

(a) 99 (b) -2 (c) 5 (d) 3 (e) 2

8. Let $A = \begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$. The complex eigenvalues of A are

(a) $\pm i$ (b) $3 \pm i$ (c) 4 (d) $\pm 3i$ (e) $1 \pm 3i$

9. For what value of h will $\mathbf{y}$ be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, if $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 6 \\ 4 \\ h \end{bmatrix}$.

(a) 3 (b) 2 (c) 4 (d) 10 (e) 6

10. Let $\mathbf{b}_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $\mathbf{c}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $\mathbf{c}_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$. Find $P = [[\mathbf{b}_1]_{\mathcal{C}}, [\mathbf{b}_2]_{\mathcal{C}}]$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

(a) $\frac{1}{7}\begin{bmatrix} -5 & 3 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 4 \\ -5 & -3 \end{bmatrix}$

(d) $\frac{1}{4}\begin{bmatrix} 1 & -5 \\ 1 & 9 \end{bmatrix}$ (e) $\begin{bmatrix} -9 & -5 \\ 1 & -1 \end{bmatrix}$

11. Find a matrix A such that $\operatorname{Col} A = \left\{ \begin{bmatrix} 9a - 8b \\ a + 2b \\ -5a \end{bmatrix} \;\middle|\; a, b \text{ in } \mathbb{R} \right\}$.

(a) $\begin{bmatrix} 9 & -8 \\ 1 & 2 \\ -5 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 9 & -8 \\ 1 & 2 \\ 0 & -5 \end{bmatrix}$

(d) $\begin{bmatrix} 9 & 1 & -5 \\ -8 & 2 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

12. Let $S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$. Then the set S

(a) is a linearly independent subset

(b) is a basis of R^3

(c) is a linearly dependent subset

(d) spans R^3

(e) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$

13. Let $A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$. Use Cramer's rule to solve $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

14. Let $\mathbb{P}_n$ be the vector space of polynomials of dgree n and let $T : \mathbb{P}_2 \to \mathbb{P}_1$ be the linear transformation given by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$. Suppose that $\mathcal{B} = \{1, t, t^2\}$ is a basis of $\mathbb{P}_2$ and $\mathcal{C} = \{1, t\}$ is a basis of $\mathbb{P}_1$.

(1) Find a matrix A such that $[T\mathbf{v}]_{\mathcal{C}} = A[\mathbf{v}]_{\mathcal{B}}$;

(2) Find $\mathrm{Null}\, A$ and $\mathrm{Col}\, A$.

15. Let $A = \frac{1}{5} \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find $\lim_{n \to \infty} A^n$.

[*Hint:* Find the diagonalization D of A and use the formula $A^n = PD^nP^{-1}$.]