Answer Key 1

MATH 20580: Introduction to Linear Algebra and Differential Equations
Practice Exam 3 *April 21, 2011*

1.	a	b	●	d	e	7.	●	b	c	d	e
2.	a	b	c	●	e	8.	a	b	c	d	●
3.	a	b	c	d	●	9.	a	b	●	d	e
4.	a	b	c	d	●	10.	a	b	c	d	●
5.	●	b	c	d	e	11.	a	●	c	d	e
6.	a	b	●	d	e	12.	●	b	c	d	e

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1. Let $A = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Find the angle between $A\mathbf{u}$ and $A\mathbf{v}$.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) 0 (e) $\frac{\pi}{6}$

2. Let $\mathbf{y} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Find the orthogonal projection of $\mathbf{y}$ onto $\mathbf{u}$.

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} 4 \\ 8 \end{bmatrix}$ (e) $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$

3. Let $\mathbf{y} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Use the fact that $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal to compute $\text{proj}_W \mathbf{y}$.

(a) $\begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$ (d) $\begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

4. Let $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$ and $W = \text{Span}\left\{\begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}\right\}$. Find the distance between $\mathbf{y}$ and W.

(a) 13 (b) 0 (c) 1 (d) 3 (e) 8

5. Let $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. Find a least-squares solution of the inconsistent system $A\mathbf{x} = \mathbf{b}$

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

6. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 7 \end{bmatrix}$, and let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Find a vector $\mathbf{u}_3$ so that the subset $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{u}_3\}$ is an orthogonal basis of W.

(a) $\begin{bmatrix} 4 \\ 2 \\ 2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -3 \\ 1 \\ -1 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 \\ 3 \\ 1 \\ 7 \end{bmatrix}$

7. Find an orthonormal basis of the subspace $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix} \right\}$.

(a) $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ -9 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ (e) $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix}$

8. Let $A = \frac{1}{7}\begin{bmatrix} 2 & 6 & 3 \\ 3 & 2 & -6 \\ 6 & -3 & 2 \end{bmatrix}$. Find A^{-1}.

(a) A^3

(b) $\begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$

(c) A^2

(d) 0

(e) $\frac{1}{7}\begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$

9. Solve the initial value problem of $ty' + 2y = 4t^2$, $y(1) = 3$.

(a) $t^2 + \frac{1}{t^2}$

(b) $t^2 - \frac{1}{t^2}$

(c) $t^2 + \frac{2}{t^2}$

(d) $2t^2 + \frac{1}{t^2}$

(e) $t^2 - \frac{2}{t^2}$

10. Solve the initial value problem $y' = 9.8 - \frac{y}{5}$, $y(0) = 50$.

(a) $1 + 49e^{-t/5}$

(b) $(9.8t + 50)e^{-t/5}$

(c) $50 - t/5$

(d) $49 + t/5$

(e) $49 + e^{-t/5}$

11. Solve the separable differential equation $y' = \dfrac{x^2}{y(1+x^3)}$.

(a) $3y^2 - \ln|1+x^3| = c$

(b) $3y^2 - 2\ln|1+x^3| = c$

(c) $2y^2 - 3\ln|1+x^3| = c$

(d) $y^2 - 2\ln|1+x^3| = c$

(e) 0

12. Suppose a sum of money is deposited in a account and left to accrue interest at an annual rate of r compounded continuously. Find the value of r so that the money in the account will double in 10 years.

(a) $\dfrac{\ln 2}{10}$ (b) 10% (c) 2 (d) $\dfrac{\ln 10}{2}$ (e) 20%

13. Find the solution of the initial value problem $\dfrac{dy}{dt} = \dfrac{1}{2}(1-y)y$, $y(0) = 4$. Then find $\lim_{t\to\infty} y(t)$.

14. Find an integrating factor for the equation $(3xy + y + 1)dx + (x^2 + xy)dy = 0$ and then solve the equation.

15. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$. Find the minimum value of $\|A\mathbf{x} - \mathbf{b}\|$ for all vectors $\mathbf{x}$ in $\mathbb{R}^2$.

[*Hint:* The least-squares solution is given by $(A^TA)^{-1}A^T\mathbf{b}$.]