Answer Key 1

MATH 20580: Introduction to Linear Algebra and Differential Equations
Practice Final *May 13, 2011*

1.	a	b	c	d	•	13.	a	•	c	d	e
2.	a	b	•	d	e	14.	a	b	c	d	•
3.	a	b	c	d	•	15.	a	b	c	•	e
4.	a	b	c	d	•	16.	a	b	c	d	•
5.	a	b	•	d	e	17.	a	b	•	d	e
6.	a	•	c	d	e	18.	a	b	c	•	e
7.	a	b	c	•	e	19.	a	b	c	•	e
8.	a	b	•	d	e	20.	•	b	c	d	e
9.	•	b	c	d	e	21.	a	b	•	d	e
10.	a	b	c	d	•	22.	a	•	c	d	e
11.	a	b	c	•	e	23.	•	b	c	d	e
12.	•	b	c	d	e	24.	a	b	•	d	e

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6.	a	b	c	d	e	18.	a	b	c	d	e
7.	a	b	c	d	e	19.	a	b	c	d	e
8.	a	b	c	d	e	20.	a	b	c	d	e
9.	a	b	c	d	e	21.	a	b	c	d	e
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1. Let $y_1(t)$ and $y_2(t)$ be a fundamental set of solutions of $y'' + y' + \dfrac{\sin t}{t} y = 0$ satisfying the initial conditions $y_1(0) = 1, y_1'(0) = 0$ and $y_2(0) = 0, y_2'(0) = 1$. Then the Wronskian $W(t) = [y_1(t)y_2'(t) - y_1'(t)y_2(t)]$ is equal to

(a) $\sin t$ (b) e^t (c) $\dfrac{\sin t}{t}$. (d) 1 (e) e^{-t}

2. Suppose $\phi(t) = A_0 + A_1 t + A_2 t^2$ is a solution to $y'' + 4y = 4t^2$ for some constants A_0, A_1, A_2. Find A_0.

(a) 4 (b) 1 (c) $-1/2$ (d) 0 (e) -1

3. Find the value of h so that the linear system $\begin{bmatrix} 1 & 5 & -3 \\ 1 & 4 & -1 \\ 2 & 7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ h \end{bmatrix}$ has a solution.

(a) 5 (b) 2 (c) 1 (d) 3 (e) -5

4. Suppose that $\phi(t) = At^s e^{-t} + B$ is a solution to $y'' - 3y' - 4y = -5e^{-t} - 4$ for some constants A, B, and s. Find A.

(a) 4 (b) -4 (c) $-2/5$ (d) -1 (e) 1

5. Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$. Find adj$\,(A)$.

(a) $\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix}$

6. Find the reduced row echelon form of $\begin{bmatrix} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{bmatrix}$.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

7. Find the integrating factor for $dx + \left(\frac{x}{y} - \sin(y) + y^2\right) dy = 0$.

(a) $\sin y$ (b) y^2 (c) 1 (d) y (e) x

8. Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \text{proj}_V \mathbf{u}$ where $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 7 \end{bmatrix}$ and $V = \text{Span}\left\{ \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\}$. Then x_1 is equal to

(a) 2 (b) 1 (c) 4 (d) 3 (e) 0

9. Which of the following is an orthonormal basis of $\mathbb{R}^2$?

(a) $\frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \frac{1}{5}\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(b) $\frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \frac{1}{5}\begin{bmatrix} -4 \\ 3 \end{bmatrix}, 0$

(c) $\frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \frac{1}{5}\begin{bmatrix} -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

(e) $\frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$. Then b_{11} is equal to

(a) 5 (b) -2 (c) -6 (d) 1 (e) 10

11. Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a solution to $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 4 & -5 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 9 \end{bmatrix}$. Then x_1 is equal to

(a) 16 (b) 9 (c) 8 (d) 29 (e) 3

12. Suppose $y_1(t) = t$ is a solution of the differential equation $t^2y'' + 2ty' - 2y = 0$. The method of reduction of order gives a second solution of the form $y_2 = v(t)y_1(t)$. Find $v(t)$.

(a) t^{-3} (b) 1 (c) $\frac{4}{t}$ (d) t^{-2} (e) t

13. Find the roots of the characteristic equation for $y'' + 100y = 0$.

(a) $-10 \pm 10i$ (b) $\pm 10i$ (c) $-100, 0$ (d) ± 10 (e) $0, 10$

14. If A is a 4×4 matrix and $\det A = 2$, then $\det(-2A)$ is

(a) 16 (b) -16 (c) -4 (d) -32 (e) 32

15. Determine which of the following form a fundamental set of solutions of linear differential equation $2t^2y'' + 3ty' - y = 0$.

(a) $t^{1/2}$, 0 (b) t, t^{-1} (c) t, 1 (d) $t^{1/2}$, t^{-1} (e) $t^{3/2}$, t

16. If $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ and $\mathbf{x} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$, then $[\mathbf{x}]_{\mathcal{B}}$ is equal to

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

17. The eigenvalues of $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ are

(a) $1, -5, 0$ (b) $-3, -5, -3$ (c) $1, -2, -2$ (d) $1, 3, 3$ (e) $1, 3, 5$

18. Let $y(t)$ be the unique solution to the initial value problem $y'' - y = 0$, $y(0) = 2$, $y'(0) = 0$. Then $y(1)$ is equal to

(a) $2e - 2$ (b) $2e^{-1}$ (c) 2 (d) $e + e^{-1}$ (e) $2e$

19. Let $y(t)$ be the unique solution to $y' + \frac{2}{t}y = 4t$ with initial condition $y(1) = 3$. Then $y(2)$ is equal to

(a) 8 (b) $\ln 2 + 2$ (c) $e^4 + 2$ (d) $4 + \frac{1}{2}$ (e) $8 + \frac{1}{4}$

20. Let $\phi(t) = v_1(t)\cos(3t) + v_2(t)\sin(3t)$ be a solution to $y'' + 9y = \frac{1}{\sin 3t}$. Then $v_2(t)$ is equal to

(a) $\frac{1}{9}\ln|\sin 3t|$ (b) $\frac{1}{3}\ln|\sin 3t|$ (c) $\frac{t}{3}$ (d) $\cos(3t)$ (e) $\frac{1}{\sin 3t}$

21. Suppose $y' = 2y^{100}(3 - y)$ and $y(0) = 5$. Find $\lim_{t\to\infty} y(t)$.
[*Hint:* You do not need to solve for $y(t)$ to find the limit.]

(a) 2 (b) 5 (c) 3 (d) 1 (e) 0

22. Let $y(t)$ be the unique solution to the initial value problem $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$. Then $y(1)$ is equal to

(a) 0 (b) $2e^{-1}$ (c) 1 (d) $e + e^{-1}$ (e) $2e$

23. Let $y(t)$ be the unique solution to the equation $y' = y^2$ with $y(0) = -1$. Then $y(1)$ is equal to

(a) $-1/2$ (b) -1 (c) -4 (d) -3 (e) 0

24. Find the determinant of $\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & 1 \\ 0 & -2 & 0 \end{bmatrix}$.

(a) 1 (b) -2 (c) 2 (d) 5 (e) 0