Lecture 4: Principles of Parallel Algorithm Design (part 3)
Exploratory Decomposition

• Decomposition according to a search of a state space of solutions

• Example: the 15-puzzle problem
  – Determine any sequence or a shortest sequence of moves that transforms the initial configuration to the final configuration.

A

B

C

D

Huarong Road Game
• **Solution algorithm**
  – Subsequent configurations are generated based on current configuration.
  – Each configuration is then explored as an independent task.
• Difference between data-decomposition and exploratory decomposition
  – Tasks induced by data-decomposition are performed entirely and each task performs useful computation towards the solution of problem.
  – Tasks induced by exploratory can be terminated before finishing as soon as desired solution is found.
• Work induced by exploratory decomposition and performed by parallel formulation can be either smaller or greater than that performed by serial algorithm
Speculative Decomposition

- This decomposition is used when a program may take one of many possible computationally significant branches depending on the output of other computations that precede it.

- While one task is performing the computation whose output is used in deciding the next configuration, other tasks can concurrently start the computations of the next stage.
  - The scenario is similar to evaluating one or more of the branches of a `switch` statement in C in parallel before the input for the `switch` is available.
Example: Speculative Decomposition

• Parallel *discrete event simulation*
  – The nodes of a directed network have input buffer of jobs. After processing the job, the node put results in the input buffer of nodes which are connected to it by outgoing edges. A node has to wait if the input buffer of one of its outgoing neighbors is full. *There is a finite number of input job types.*

![Diagram](image)

• Inherently sequential problem
• Can be improved by starting simulating a subpart of the network, each assume one of several possible inputs to that stage (*overlapping different computations*).
Hybrid Decomposition

• Use several decomposition methods together
• Example: finding the minimum of any array of size 16 using 4 tasks.
Characteristics of Tasks

Key characteristics of tasks influencing choice of mapping and performance of parallel algorithm:

1. Task generation
   - Static or dynamic generation
     - Static: all tasks are known before the algorithm starts execution. Data or recursive decomposition often leads to static task generation.
       Ex. Matrix-multiplication. Recursive decomposition in finding min. of a set of numbers.
     - Dynamic: the actual tasks and the task-dependency graph are not explicitly available a priori. Recursive, exploratory decomposition can generate tasks dynamically.
       Ex. Recursive decomposition in Quicksort, in which tasks are generated dynamically.

2. Task sizes
   - Amount of time required to compute it: uniform, non-uniform

3. Knowledge of task sizes
   - Ex. Size of task in 15-puzzle problem is unknown.

4. Size of data associated with tasks
   - Data associated with the task must be available to the process performing the task. The size and location of data may determine the data-movement overheads.
Characteristics of Task Interactions

1) Static versus dynamic
   – Static: interactions are known prior to execution.

2) Regular versus irregular
   – Regular: interaction pattern can be exploited for efficient implementation.

3) Read-only versus read-write

4) One-way versus two-way
Static vs. Dynamic Interactions

• Static interaction
  – Tasks and associated interactions are predetermined: task-interaction graph and times that interactions occur are known: matrix multiplication
  – Easy to program

• Dynamic interaction
  – Timing of interaction or sets of tasks to interact with cannot be determined prior to the execution.
    Ex. Puzzle game. The tasks has exhausted its work can pick up an unexplored state from the queue of another busy task and start exploring it.
  – Difficult to program using massage-passing; Shared-memory space programming may be simple
Regular vs. Irregular Interactions

• Regular interactions
  – Interaction has a spatial structure that can be exploited for efficient implementation: ring, mesh
    Example: Explicit finite difference for solving PDEs. Image dithering.

• Irregular Interactions
  – Interactions has no well-defined structure
    Example: Sparse matrix-vector multiplication
Static regular interaction for image dithering
Read-Only vs. Read-Write Interactions

• Read-only interactions
  – Tasks only require read-only interactions
  – Example: matrix-matrix multiplication

\[
\begin{pmatrix}
A_{1,1} & A_{1,2} \\
A_{2,1} & A_{2,2}
\end{pmatrix}
\cdot
\begin{pmatrix}
B_{1,1} & B_{1,2} \\
B_{2,1} & B_{2,2}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
C_{1,1} & C_{1,2} \\
C_{2,1} & C_{2,2}
\end{pmatrix}
\]

Task 1: \( C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1} \)
Task 2: \( C_{1,2} = A_{1,1}B_{1,2} - A_{1,2}B_{2,2} \)
Task 3: \( C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1} \)
Task 4: \( C_{2,2} = A_{2,1}B_{1,2} - A_{2,2}B_{2,2} \)

• Read-write interactions
  – Multiple tasks need to read and write on some shared data
One-Way vs. Two-Way Interactions

• One-way interactions
  – One of a pair of communicating tasks initiates the interaction and completes it without interrupting the other one.
  – Example: read-only can be formulated as one-way

• Two-way interactions
  – Both tasks involve in interaction
  – Example: read-write can be formulated as two-way