MATH 20550: Calculus III
Practice Exam 1

Multiple Choice Problems

1. Find an equation for the line through the point $(3,-1,2)$ and perpendicular to the plane $2x-y+z+10=0$.

   (a) $\frac{x-3}{2}=\frac{y+1}{-1}=z-2$ (b) $\frac{x+3}{2}=\frac{y-1}{-1}=z-2$
   (c) $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-2}{2}$ (d) $3x-y+2z+10=0$
   (e) $3x-2y+z+10=0$

2. Find an equation of the plane that passes through the point $(1,2,3)$ and parallel to $x-y+z=100$.

   (a) $x-y+z-2=0$ (b) $x-y+z+2=0$ (c) $x+2y+3z=100$
   (d) $x-1=2-y=z-3$ (e) $x-1=\frac{y+1}{2}=\frac{z-1}{3}$.

3. Find the distance between the point $(-1,-1,-1)$ and the plane $x+2y+2z-1=0$.

   (a) 2 (b) 0 (c) 6 (d) -2 (e) -6

4. Find values of b such that the vectors $<-11,b,2>$ and $<b,b^2,b>$ are orthogonal

   (a) 0, 3, -3 (b) 0, 11, 3 (c) 0, -11, 2
   (d) 0, 2, -2 (a) 0, 11, 2

5. Find the area of the triangle with vertices at the points $(0,0,0)$, $(1,0,-1)$ and $(1,-1,2)$.

   (a) $\frac{\sqrt{11}}{2}$ (b) $\sqrt{11}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{6}}{2}$ (e) 1

6. Which vector is always orthogonal to $\mathbf{b}-proj_{\mathbf{a}}\mathbf{b}$.

   (a) $\mathbf{a}$ (b) $\mathbf{b}$ (c) $\mathbf{a}-\mathbf{b}$ (d) $|\mathbf{a}|\mathbf{b}$ (e) $proj_{\mathbf{b}}\mathbf{a}$

7. Find the parametric equations of the intersection of the planes $x-z=0$ and $x-y+2z+3=0$.

   (a) The line given by $x=-t$, $y=3-3t$ and $z=-t$.
   (b) The line given by $x=-2-t$, $y=1-3t$ and $z=-t$.
   (c) The line given by $x=1+t$, $y=6-t$ and $z=1+2t$.
   (d) The plane $3x+3y-3z+3=0$.
   (e) The line given by $x=1+t$, $y=6$ and $z=1-t$.

8. Find an equation for the normal plane to the vector function

   $$\mathbf{r}(t)=\langle e^{t-1},t^2,\cos(1-t)\rangle$$

   when $t=1$.

(a) $x + 2y - 3 = 0$.

(b) $x + y + z - 3 = 0$.

(c) $x = 1 + t,\ y = 1 + 2t,\ z = 1$.

(d) Does not exist since $\mathbf{r}'(0) = \mathbf{0}$.

(e) $x + 2y - 3 = 0$ and $z = 0$.

9. The equation of the sphere with center $(4, -1, 3)$ and radius $\sqrt{5}$ is

(a) $(x-4)^2+(y+1)^2+(z-3)^2 = 5$ (b) $(x-4)^2+(y+1)^2+(z-3)^2 = 25$
(c) $(x-4)^2+(y+1)^2+(z-3)^2 = \sqrt{5}$ (d) $(x+4)^2+(y-1)^2+(z+3)^2 = 5$
(e) $(x-4)^2 + (y-1)^2 + (z-3)^2 = 5$

10. Suppose a particle's position at time $t > 0$ is described by

$$\mathbf{r}(t) =< t^2,\ -2t,\ \ln t > .$$

Find the tangential component a_T and normal component a_N of the acceleration at $t = 1$.

(a) $a_T = 1,\ \ a_N = 2$

(b) $a_T = 2,\ \ a_N = 1$

(c) $a_T = 1,\ \ a_N = -2$

(d) $a_T = 1,\ \ a_N = \sqrt{5}$

(e) $a_T = \sqrt{5},\ \ a_N = 1$

11. Find the volume of the parallelepiped determined by the vectors $< 1, 2, 7 >$, $< 0, -3, 4 >$, and $< 0, 0, 6 >$.

(a) 18 (b) 12 (c) 0 (d) 16 (e) 20

12. Let

$$f(x, y) = e^{\sin x} + x^5 y + \ln(1 + y^2).$$

Find $\frac{\partial^2 f}{\partial x \partial y}$.

(a) $5x^4$ (b) $\frac{2y}{1+y^2}$ (c) $20x^3 y$ (d) $e^{\sin x}\cos x$
(e) $e^{\sin x}\cos x + x^5 + \frac{2y}{1+y^2}$

Partial Credit Problems

13. Find the length of the curve

$$\mathbf{r}(t) =< -\ln t,\ t^2,\ 2t >, \qquad 1 \le t \le e.$$

14. Find the unit normal vector as a function of t, $\mathbf{N}(t)$, where $\mathbf{r}(t) = \langle 3t, 4\cos(t), 4\sin(t)\rangle$.

15. A projectile is fired with initial speed 2 m/s at an elevation angle $\frac{\pi}{6}$ from a height of 4 meters above the ground. Assume the only force acting on the object is gravity. Find the horizontal range (i.e. the horizontal distance travelled before landing) of the projectile. For the simplicity of computation, take the constant acceleration due to gravity $g = 10\ m/s^2$.