MATH 20550: Calculus III
Practice Exam 2

Multiple Choice Problems

1. Find the directional derivative of $f(x,y) = e^{x\sin(y)}$ at $P(1,0)$ in the direction of $Q(-2,4)$.

   (a) 4/5 (b) -3/5 (c) -2 (d) 0 (e) 4

2. Suppose the line $2x - y + 1 = 0$ is tangent to the level curve $f(x,y) = 2$ at the point $(0,1)$. Determine which of the vectors below is parallel to $\nabla f(0,1)$.

   (a) $\langle -2, 1\rangle$ (b) $\langle 1, 2\rangle$ (c) $\langle 2, 1\rangle$ (d) $\langle 1, -2\rangle$ (e) $\langle 1, 1\rangle$

3. Let $f(x,y) = 8x^3 + 6xy + y^3$. Which of the following statements is true about the critical points of f?

   (a) $(0,0)$ is a saddle point and $(-1/2,-1)$ is a local maximum.
   (b) $(0,0)$ is a saddle point and $(-1/2,-1)$ is a local minimum.
   (c) $(0,0)$ is a local maximum and $(-1/2,-1)$ is a local minimum.
   (d) $(0,0)$ is a local minimum and $(-1/2,-1)$ is a local maximum.
   (e) $(0,0)$ is a local maximum and $(-1/2,-1)$ is a saddle point.

4. Let C be the curve of intersection of the surfaces $x^2 + y^2 + z^2 = 1$ and $xy + z^2 = 0$. Determine which of the following systems of equations must be solved to find the point(s) on C closest to $(1,1,1)$ using Lagrange multipliers.

   (a)
   $$\begin{gathered} 2(x-1) = 2\lambda x + \mu y \\ 2(y-1) = 2\lambda y + \mu x \\ (z-1) = \lambda z + \mu z \\ x^2 + y^2 + z^2 = 1 \\ xy + z^2 = 0 \end{gathered}$$

   (b)
   $$\begin{gathered} 2x = 2\lambda(x-1) + \mu y \\ 2y = 2\lambda(y-1) + \mu x \\ z = \lambda(z-1) + \mu z \\ x^2 + y^2 + z^2 = 1 \\ xy + z^2 = 0 \end{gathered}$$

   (c)
   $$\begin{gathered} (x-1) = \lambda x \\ (y-1) = \lambda y \\ (z-1) = \lambda z \\ x^2 + y^2 - xy = 1 \end{gathered}$$

   (d)
   $$\begin{gathered} 2(x-1) = \lambda y \\ 2(y-1) = \lambda x \\ (z-1) = \lambda z \\ x^2 + y^2 - xy = 1 \end{gathered}$$

   (e)
   $$\begin{gathered} 2(x-1) = \lambda(2x - y) \\ 2(y-1) = \lambda(2y - x) \\ 2(z-1) = \lambda(x+y) \\ x^2 + y^2 - xy = 1 \end{gathered}$$

5. Reverse the order of integration in the double integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$.

(a) $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\ dx\ dy$ (b) $\int_0^{\sqrt{1-x^2}}\int_{-1}^{1} f(x,y)\ dx\ dy$
(c) $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^1 f(x,y)\ dx\ dy$ (d) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\ dx\ dy$
(e) $\int_{-1}^{1}\int_0^{\sqrt{1-y^2}} f(x,y)\ dx\ dy$

6. Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{2x^4y^2}{x^4+3y^4}.$$

(a) 0 (b) 2 (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (e) Doesn't exist.

7. Let

$$f(x,y) = e^{\sin x} + x^5 y + \ln(1+y^2).$$

Find $\frac{\partial^2 f}{\partial x \partial y}$.

(a) $5x^4$ (b) $\frac{2y}{1+y^2}$ (c) $20x^3y$ (d) $e^{\sin x}\cos x$
(e) $e^{\sin x}\cos x + x^5 + \frac{2y}{1+y^2}$

8. Find $\frac{\partial z}{\partial x}$ if $yz^3 + ze^{xy} = \cos x$.

(a) $\frac{-\sin x - yze^{xy}}{3yz^2+e^{xy}}$ (b) $\frac{y\cos x - ze^{xy}}{xz^2 - ye^{xy}}$ (c) $\frac{\cos x - e^{xy}}{z^2 - ye^{xy}}$
(d) $\frac{\sin x}{3x^2 - e^{xy}}$ (e) $\frac{\cos x + e^{xy}}{z^2 - 4ye^{xy}}$

9. What is the direction in which the function

$$f(x,y) = yx^2 - \frac{x}{y^2}$$

increases most rapidly at the point $(-2,1)$.

(a) $< -5, 0 >$ (b) $< 3, 2 >$ (c) $< 0, 0 >$ (d) $< 2, 2 >$
(e) $< 3, 1 >$

10. Given that $f_x(1,2) = 0$, $f_y(1,2) = 0$, $f_{xx}(1,2) = -4$, $f_{yy}(1,2) = -9$, and $f_{yx}(1,2) = 6$, then

(a) There is not enough information given to determine.
(b) $f(1,2)$ is a local max.
(c) $f(1,2)$ is a local min.
(d) The point $(1,2)$ is a saddle point.
(e) $f(1,2)$ is not defined.

11. Find the average value of function $f(x,y) = 2xy$ over the rectangle

$$R = \{(x,y) \mid 0 \le x \le 3,\ \ 0 \le y \le 2\}.$$

(a) 3 (b) 18 (c) 9 (d) 6 (e) 12

12. Compute

$$\int_0^1 \int_0^x \frac{1}{1+x^2}\, dy\, dx.$$

(a) $\frac{\ln 2}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{\ln 5}{2}$ (e) $\ln 3 - 1$.

Partial Credit Problems

13. Find the points on the surface defined by $(x-y)^2 + y^2 + (y+z)^2 = 1$ at which the tangent plane is parallel to the xz-plane.

14. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x + 1$ on the unit disk $x^2 + y^2 \leq 1$.

15. Let D be the triangular region in the plane with vertices $(0, 0)$, $(2, 0)$, and $(1, 2)$. Compute $\iint_D x dA$.