MATH 20550: Calculus III
Practice Exam 3

Multiple Choice Problems

1. Find moment about the yz plane of the solid E bounded by the parabolic cylinder $z = 1-y^2$ and the planes $x+z = 1$, $x = 0$ and $z = 0$ with density $\rho(x, y, z)$.

   (a) $\int_{-1}^{1}\int_{0}^{1-y^2}\int_{0}^{1-z} x\rho(x,y,z)dxdzdy$ (b) $\int_{-1}^{1}\int_{0}^{1-y^2}\int_{0}^{1-z} yz\rho(x,y,z)dxdzdy$
   (c) $\int_{0}^{1}\int_{0}^{1-y^2}\int_{0}^{1-z} x\rho(x,y,z)dxdzdy$ (d) $\int_{-1}^{1}\int_{0}^{1-z}\int_{0}^{1-y^2} x\rho(x,y,z)dzdxdy$
   (e) $\int_{0}^{1}\int_{0}^{1-y^2}\int_{0}^{1-z} y\rho(x,y,z)dxdzdy$

2. Determine which of the following integrals gives the volume of the solid region bounded by the paraboloid $z = 3y^2 + 3x^2$ and the cone $z = 4 - \sqrt{x^2 + y^2}$.

   (a) $\int_{0}^{2\pi}\int_{0}^{1}\int_{3r^2}^{4-r} rdzdrd\theta$ (b) $\int_{0}^{2\pi}\int_{0}^{1}\int_{0}^{4-r} 3r^2dzdrd\theta$
   (c) $\int_{0}^{2\pi}\int_{0}^{\pi}\int_{3r^2}^{4-r} r^2 sin\theta drdzd\theta$ (d) $\int_{0}^{2\pi}\int_{0}^{3}\int_{0}^{4-r^2} rdzdrd\theta$
   (e) $\int_{0}^{2\pi}\int_{0}^{4}\int_{0}^{3} rsin\theta drdzd\theta$

3. Evaluate the line integral with respect to arc length

$$\int_C x \quad ds$$

   where C is the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

   (a) $(5\sqrt{5} - 1)/12$ (b) $(2\sqrt{2} - 1)/6$ (c) 0
   (d) $\sqrt{5}/2$ (e) 1

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(t) = 3\sqrt[5]{xy}\,\mathbf{i} - \mathbf{j}$ and C is the curve $y^2 = x^3$ from $(0, 0)$ to $(1, 1)$.

   (a) 1 (b) 2 (c) 0 (d) -1 (e) 3

5. Find the area inside the cardioid $r = 2 + \cos(\theta)$.

   (a) $\frac{9\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 6π (d) $\frac{15\pi}{2}$ (e) 3π

6. Consider the loop (one leaf) of the 4-leaf rose $r = \cos 2\theta$ which is entirely contained in the first and fourth quadrant. If this region has density $\rho(x, y) = x^2 + y^2$ then which of the following integrals is the moment about the y-axis.

   (a) $\int_{-\pi/4}^{\pi/4}\int_{0}^{\cos 2\theta} r^4 \cos\theta\, dr\, d\theta$ (b) $\int_{-\pi/2}^{\pi/2}\int_{0}^{\cos 2\theta} r^4 \cos\theta\, dr\, d\theta$
   (c) $\int_{-\pi/4}^{\pi/4}\int_{0}^{\cos 2\theta} r^3 \cos\theta\, dr\, d\theta$ (d) $\int_{-\pi/2}^{\pi/2}\int_{0}^{\cos 2\theta} r^3 \cos\theta\, dr\, d\theta$
   (e) $\int_{-\pi/4}^{\pi/4}\int_{0}^{\cos 2\theta} xr^3\, dr\, d\theta$

7. What is $\int_C (\nabla f) \cdot d\mathbf{r}$ if $f(x, y, z) = xy^2 - xze^{yz}$ and C is a curve from $(1, 0, 1)$ to $(3, 2, 0)$.

(a) 13 (b) 10 (c) 0 (d) -5 (e) 21

8. Let E be the solid region bounded by $z = x^2 + y^2$ and $z = 3 - 2x^2 - 2y^2$. Suppose the volume of E is $\frac{3\pi}{2}$ and the density of E is constant. Find the center of mass of E.

(a) $(0, 0, \frac{4}{3})$ (b) $(-\frac{1}{2}, \frac{1}{2}, 3)$ (c) $(0, 0, \sqrt{2})$ (d) $(0, 0, \frac{3}{2})$ (e) $(0, 0, 1)$

Partial Credit Problems

9. Under the change of variables $x = s^2 - t^2$, $y = 2st$, the quarter circular region in the st-plane given by $s^2 + t^2 \leq 1$ is mapped onto a certain region D of the xy-plane. Evaluate

$$\int\int_D \frac{dxdy}{\sqrt{x^2 + y^2}}.$$

10. Let $\mathbf{F} = z^2\mathbf{i} + z \exp yz\mathbf{j} + (2xz + \cos z + y \exp yz)\mathbf{k}$. Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

11. Find the volume of the solid under the surface $z = \sin(x^2 + y^2)$ and above the annulus $D = \{(x, y) \mid \frac{\pi}{4} \leq x^2 + y^2 \leq \frac{\pi}{2}\}$.

12. A lamina of uniform density occupies the region D bounded by the parabola $y = 1 - x^2$ and the x-axis. Find its center of mass.