

Homework 4: Context-free languages

Theory of Computing (CSE 30151), Spring 2024

Due: 2024-02-23 5pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
 - If you're making a complete submission, name it *netid-hw4.pdf*, where *netid* is replaced with your NetID.
 - If you're submitting some problems now and want to submit other problems later, name it *netid-hw4-part123.pdf*, where 123 is replaced with the problem number(s) you are submitting at this time.
- Submit your PDF file(s) in Canvas.

Problems (10 points each)

1. **Arithmetic expressions.** Consider the grammar G_4 (page 105) for arithmetic expressions, with start symbol E :

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid a \mid b \mid c$$

- (a) [cf. Exercise 2.1] Give derivations for the following strings. You may write them either as a sequence of rewrites ($E \Rightarrow \dots$) or as a tree.
 - i. $a + b + c$
 - ii. $a * b + c$
 - iii. $a * (b + c)$

(b) Modify G_4 to allow an exponentiation operator \uparrow .

- It should have *higher precedence* than multiplication; that is, in the derivation of the string $\mathbf{a * b \uparrow c}$, there should be a nonterminal that rewrites to $\mathbf{b \uparrow c}$, and there should not be a nonterminal that rewrites to $\mathbf{a * b}$.
- It should be (unlike $*$ and $+$) *right-associative*; that is, in the derivation of the string $\mathbf{a \uparrow b \uparrow c}$, there should be a nonterminal that rewrites to $\mathbf{b \uparrow c}$, and there should not be a nonterminal that rewrites to $\mathbf{a \uparrow b}$.

2. Write both a PDA *and* a CFG for the language (page 80):

$$C = \{w \in \{0, 1\}^* \mid w \text{ has an equal number of 0s and 1s}\}.$$

Please include a brief explanation of why they work. (If you convert a PDA to a CFG or vice versa, that suffices as an explanation.)

3. [Exercise 2.6b] Write both a PDA *and* a CFG for the language

$$L_3 = \overline{\{0^n 1^n \mid n \geq 0\}}.$$

For example, $000111 \notin L_3$. Please include a brief explanation of why they work. (If you convert a PDA to a CFG or vice versa, that suffices as an explanation.)

Hint: First prove that this is equal to $\{0^m 1^n \mid m \neq n\} \cup \overline{0^* 1^*}$.