Homework 5: Non-context-free languages and Turing machines

CSE 30151 Fall 2020

Due Friday, 2020/10/09 at 5:00pm

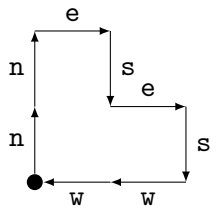
Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw5.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw5-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

1. **Non-closure properties of CFLs**
   (a) [Exercise 2.2a] Use the languages
   $$A = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n \mid m, n \geq 0\}$$
   $$B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid m, n \geq 0\}$$
   to prove that context-free languages are *not* closed under intersection.
   (b) [Exercise 2.2b] Use (a) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.
2. **There and back again.** Imagine a robot turtle that you can give instructions `n` (go north 1 inch), `s` (go south 1 inch), `w` (go west 1 inch), `e` (go east 1 inch). A program is a string of instructions.

Let C be the set of programs that make the turtle return to its starting point. For example, `nneseww` is in C, as shown in this picture:



(a) Prove that C is not context-free.

(b) Write a **formal description** of a Turing machine that decides C.

3. The Fibonacci numbers are defined by the recurrence $f_1 = 0, f_2 = 1, f_i = f_{i-2} + f_{i-1}$. Define

$$F = \{1^n \mid n \text{ is a Fibonacci number}\}.$$

(a) Prove that F is not context-free.

(b) Write an **implementation-level description** of a Turing machine that decides F.