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Reliable Decentralized Supervisory Control of Discrete Event Systems with Communication Delays

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Abstract— **The reliable decentralized supervisory control of discrete event systems (DESs) with communication delays is investigated in this paper. For a system equipped with n local supervisors, we formalize the notion of k-reliable ($1 \leq k \leq n$) decentralized supervisor under communication delays, in which some local supervisors are allowed to fail and the system can achieve exactly the specification under any k local supervisors. In particular, necessary and sufficient conditions for the existence of a k-reliable decentralized supervisor under communication delays are presented by means of the modified controllability and reliable delay-coobservability. These results can be reduced to those in [10] if the communication delays are negligible. Moreover, the results of [8] can be regarded as a special case of the proposed k-reliable decentralized control with $k = n$.**

Index Terms— Discrete event systems, decentralized supervisory control, communication delays, reliable delay-coobservability, reliable control.

I. INTRODUCTION

Recently, the reliable decentralized supervisory control of DESs began to attract more and more researchers' attention and made remarkable progresses [10] [11] [12]. Roughly speaking, for a system controlled by n local supervisors, a decentralized supervisor is called *k-reliable* ($1 \leq k \leq n$) if it achieves the given specification under possible failures of any no more than $n - k$ local supervisors. In [10], some necessary and sufficient conditions for the existence of a k-reliable decentralized supervisor for a closed language specification were presented. The results of [10] were extended to non-closed marked language specifications in [11] by the same group. Furthermore, the authors further considered the reliable decentralized supervisory control problem of DESs with the conjunctive and disjunctive fusion rules in [12].

However, all of the results on reliable decentralized supervisory control in [10] [11] [12] are based on the assumption that the control action for an observed event sequence is applied to a system without any communication delay. As a matter of fact, this assumption is only realistic [8]. In many real-world situations, there may exist non-negligible delays in sensing, communicating and/or actuating [8]. It may take time to process and transmit messages through communication networks that link the sensors, supervisors and actuators. These delays experienced in real applications should be considered in the supervisor design process otherwise the designed control law may fail in face of communication delays. Therefore, the communication delays for DESs have received more and more considerations in the literature. For example, the problem of designing embedded decentralized supervisors under delay was considered in [6]. Barrett and Lafortune [1] proposed a novel framework for analysis and synthesis of decentralized supervisory control with communication delays. Tripakis [13] investigated the decentralized supervisory control of DESs with bounded or unbounded communication delays. Park and Cho [7] studied the delay-robust supervisory control in a centralized framework, and then extended it to the decentralized framework based on conjunctive and permissive decision structure [8].

In this paper, we investigate the reliable decentralized supervisory control of DESs with communication delays, in which some uncontrollable events can unexpectedly occur before a proper control action is actually performed. In particular, the problem studied here can be formulated as:

> *For a given plant controlled by n local supervisors under communication delays, find the conditions for the existence of a reliable decentralized supervisor such that the decentralized supervisor achieves exactly the desired specification under possible failure of some local supervisors.*

Firstly, the notion of k-reliable decentralized supervisor is formalized for DESs with communication delays. Then the concepts of $\widetilde{\Sigma}_{uc}$-controllability and k-reliably delay-$\widetilde{\Sigma}_c$-coobservability are introduced. In particular, we present the necessary and sufficient conditions for the existence of a k-reliable decentralized supervisor under communication delays. It is worth noting that the results of this paper can be reduced to those in [10] [11] [12] if the communication delays are negligible. Moreover, the decentralized supervisory control under communication delays proposed in [8] can be regarded as a special case of k-reliable decentralized control investigated in this paper with $k = n$.

II. PRELIMINARIES

Consider a DES modeled by an automaton

$$G = (Q, \Sigma, \delta, q_0, Q_m), \tag{1}$$

where Q is the set of states, Σ is the finite set of events, $\delta : Q \times \Sigma \to Q$ is the transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states. Let Σ^* denote the set of all finite strings over Σ, including the empty string ϵ. The transition function δ can be extended to domain

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$Q \times \Sigma^*$ in the following recursive manner: $\delta(q, \epsilon) = q$ and $\delta(q, s\sigma) = \delta(\delta(q, s), \sigma)$ for all $s \in \Sigma^*$ and $\sigma \in \Sigma$.

A subset of Σ^* is customarily called a *language*. The language generated by G, denoted by $L(G)$, is defined by

$$L(G) = \{s \in \Sigma^* : \delta(q_0, s) \text{ is defined}\}, \quad (2)$$

and the language marked by G is defined as

$$L_m(G) = \{s \in L(G) : \delta(q_0, s) \in Q_m\}. \quad (3)$$

For a language $K \subseteq \Sigma^*$, we denote the set of all prefixes of strings in K as $\overline{K}$, i.e.,

$$\overline{K} = \{s \in \Sigma^* : st \in K \text{ for some } t \in \Sigma^*\}. \quad (4)$$

K is called to be *prefix-closed* if $K = \overline{K}$; and K is called to be $L_m(G)$*-closed* if $K = \overline{K} \cap L_m(G)$.

In the decentralized control architecture [2] [3], a system G is jointly controlled by n local supervisors $S_{P_1}, S_{P_2}, \cdots, S_{P_n}$ according to the fusion rule on the local decision actions, and each local supervisor can observe the locally observable events and can control the locally controllable events. Denote $\Sigma_{i,c}$ and $\Sigma_{i,uc}$ as the sets of locally controllable and uncontrollable events, respectively; $\Sigma_{i,o}$ and $\Sigma_{i,uo}$ as the sets of locally observable and unobservable events, respectively, where $i \in I = \{1, 2, \cdots, n\}$. The *projection* $P_i : \Sigma^* \to \Sigma_{i,o}^*$ is defined inductively as $P_i(\epsilon) = \epsilon$, and for $\sigma \in \Sigma$ and $s \in \Sigma^*$,

$$P_i(s\sigma) = \begin{cases} P_i(s)\sigma, & \text{if } \sigma \in \Sigma_{i,o}, \\ P_i(s), & \text{otherwise.} \end{cases} \quad (5)$$

The sets of globally controllable and globally observable events are respectively defined as $\Sigma_c = \cup_{i \in I} \Sigma_{i,c}$ and $\Sigma_o = \cup_{i \in I} \Sigma_{i,o}$, the sets of globally uncontrollable and unobservable events are defined respectively as $\Sigma_{uc} = \Sigma - \Sigma_c$ and $\Sigma_{uo} = \Sigma - \Sigma_o$.

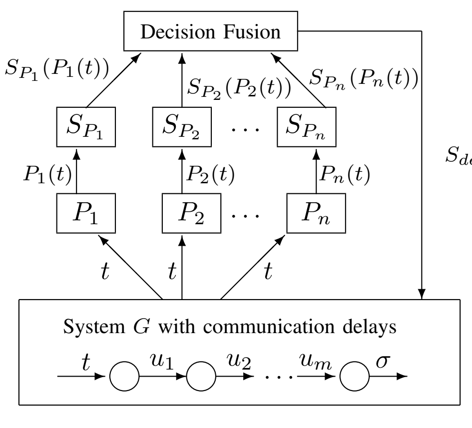

Fig. 1. Decentralized supervisory control under delays.

In most practical applications of networked dynamical systems, there are unavoidable delays in processing and transmission packets between sensors, controllers and actuators. As a result, a series of uncontrollable events may occur before a proper control action is actually applied to the system. This scenario is illustrated in Fig.1, where a proper control action is performed after a subsequent occurrence of some uncontrollable events $u_1, u_2, \cdots, u_m$. Motivated by the work in [8], we consider the *reliable decentralized supervisory control* of DESs with communication delays in this paper.

III. An Approach to Synthesize Part of Local Supervisors under Communication Delays

In order to illustrate the reliable decentralized supervisory control of DESs with communication delays, in this section, we present an approach to synthesize a part of local supervisors, and then investigate some main properties of the synthesis, which will be used to deduce the conditions of the existence of reliable decentralized supervisor.

For $\sigma \in \Sigma_c$, denote

$$In(\sigma) = \{i \in I : \sigma \in \Sigma_{i,c}\},$$

where $I = \{1, 2, \cdots, n\}$. For $A \in 2^I$, define $\Sigma_{A,c} = \cup_{i \in A} \Sigma_{i,c}$ and $\Sigma_{A,uc} = \Sigma - \Sigma_{A,c}$.

Definition 1: Let $i \in I$ and $K \subseteq L(G)$. *The local supervisor under communication delays* is defined as a function

$$S_{P_i} : P_i(\Sigma^*) \to \Gamma = \{\gamma \in 2^\Sigma : \Sigma_{i,uc} \subseteq \gamma\},$$

where

$$\begin{aligned} S_{P_i}(P_i(s)) \quad = \quad & \{\sigma \in \Sigma_{i,c} : (\exists s' \in \overline{K})(\exists u \in \Sigma_{uc}^*) \\ & [P_i(s) = P_i(s') \wedge s'u\sigma \in \overline{K}]\} \\ & \cup (\Sigma_c - \Sigma_{i,c}) \cup \Sigma_{uc}. \end{aligned} \quad (6)$$

Intuitively, $S_{P_i}(P_i(s))$ represents the set of events enabled by the local supervisor S_{P_i} after the occurrence of the observation $P_i(s)$, in which the uncontrollable events may occur before the control action is performed.

Remark 1: Definition 1 means that $S_{P_i}(P_i(s))$ consists of not only the locally uncontrollable events but also the locally controllable legal events defined after an uncontrollable event sequence along an estimation s' of the string s. In particular, if the term 'u' is removed from the definition, then it is consistent with the standard definition of $S_{P_i}(P_i(s))$ proposed in [2].

Definition 2: Let S_{P_i} $(i = 1, 2, \cdots, n)$ be the local supervisors under communication delays. For $A \in 2^I$, the *A-decentralized supervisor under communication delays*, denoted by S_A, is defined as

$$S_A(s) = (\bigcap_{i \in A} S_{P_i}(P_i(s))) \cup \Sigma_{A,uc}. \quad (7)$$

Remark 2: If $A = I$ and the communication delays of $S_{P_i}(P_i(s))$ are negligible, then Definition 2 coincides with that in [2] (page 210 of [2]).

Definition 3: The language generated by S_A, denoted by $L(G, S_A)$, is defined recursively in the usual manner: $\epsilon \in L(G, S_A)$ and for any $s \in \Sigma^*$ and $\sigma \in \Sigma$,

$$s\sigma \in L(G, S_A) \Leftrightarrow s \in L(G, S_A), s\sigma \in L(G), \sigma \in S_A(s). \quad (8)$$

The marked language $L_m(G, S_A) = L(G, S_A) \cap L_m(G)$.

Proposition 1: Let $A, B \in 2^I$ and $K \subseteq L(G)$. If $L(G, S_A) = \overline{K} = L(G, S_B)$, then $L(G, S_{A \cup B}) = \overline{K}$.

Proof: It can be verified directly by induction on the length of the strings in $L(G, S_A)$, $L(G, S_B)$ and $\overline{K}$.

Next, we present the condition of the existence of A-decentralized supervisor S_A satisfying $L(G, S_A) = \overline{K}$ by the following notions of $\Sigma_{A,uc}$-controllability and delay-$\Sigma_{A,c}$-coobservability.

Definition 4: Let $A \in 2^I$. A language $K \subseteq L(G)$ is said to be $\Sigma_{A,uc}$-*controllable* if $\overline{K}\Sigma_{A,uc} \cap L(G) \subseteq \overline{K}$.

Definition 5: Let $A \in 2^I$. A language $K \subseteq L(G)$ is said to be *delay-*$\Sigma_{A,c}$*-coobservable*, if for any $su \in \overline{K}$ and any $\sigma \in \Sigma_{A,c}$ satisfying $u \in \Sigma_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, then there is $i \in A$ such that $\sigma \in \Sigma_{i,c}$ and

$$(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset. \tag{9}$$

The definition of delay-$\Sigma_{A,c}$-coobservability is a generalization of that corresponding definition in the absence of delays introduced in [10] (page 662 of [10]). In order to illustrate the concept, we provide the following example.

Example 1: Consider a DES G modeled by an automaton shown in Fig. 2, where $n = 3$ (i.e., $I = \{1, 2, 3\}$) and

$$\Sigma_{1,o} = \{a, c, u_1\}, \quad \Sigma_{2,o} = \{b, c, u_2\}, \quad \Sigma_{3,o} = \{a, b\};$$
$$\Sigma_{1,c} = \{a, c\}, \quad \Sigma_{2,c} = \{b, c\}, \quad \Sigma_{3,c} = \{a, b\}.$$

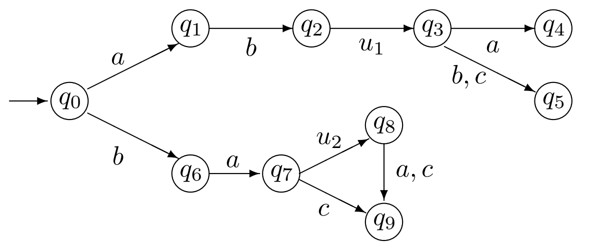

Fig. 2. A DES G.

Consider language

$$K = \overline{abu_1(b + c) + bau_2(a + c) + bac}.$$

Take $A = \{1, 2\}$, then $\Sigma_{A,c} = \{a, b, c\}$, $\Sigma_{A,uc} = \{u_1, u_2\}$, and $L(G) - \overline{K} = \{abu_1a\}$. For $s = ab$, $u = u_1$ and $\sigma = a$, we have $su \in \overline{K}$, $\sigma \in \Sigma_{A,c}$, $u \in \Sigma_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, but $\sigma \notin \Sigma_{2,c}$ and $bau_2a \in (P_1^{-1}P_1(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K}$. That is, there is not $i \in A = \{1, 2\}$ such that $\sigma \in \Sigma_{i,c}$ and $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset$. Therefore, K is not delay-$\Sigma_{A,c}$-coobservable when $A = \{1, 2\}$.

However, if we take $A = \{1, 2, 3\}$, then K is delay-$\Sigma_{A,c}$-coobservable. It is because that only one case that $s = ab$, $u = u_1$ and $\sigma = a$ satisfies the conditions of $su \in \overline{K}$, $\sigma \in \Sigma_{A,c}$, $u \in \Sigma_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, furthermore, $\sigma \in \Sigma_{i,c}$ and $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset$ hold when $i = 3$.

In the following, we present a necessary and sufficient condition for the existence of A-decentralized supervisor.

Theorem 1: Let $A \in 2^I$ and $K \subseteq L(G)$. There is an A-decentralized supervisor under communication delays S_A such that $L(G, S_A) = \overline{K}$ if and only if K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable.

Proof: ($\Rightarrow$) First we prove that K is $\Sigma_{A,uc}$-controllable. For any $s \in \overline{K}$ and any $\sigma \in \Sigma_{A,uc}$ that $s\sigma \in L(G)$, from $L(G, S_A) = \overline{K}$ and Eq. (7), $s \in L(G, S_A)$ and $\sigma \in S_A(s)$. By Definition 3, we have $s\sigma \in L(G, S_A)$, i.e., $s\sigma \in \overline{K}$. Therefore, $\overline{K}\Sigma_{A,uc} \cap L(G) \subseteq \overline{K}$.

Next, we check that K is delay-$\Sigma_{A,c}$-coobservable by contradiction. Assume that there is $su \in \overline{K}$ and $\sigma \in \Sigma_{A,c}$ satisfying $u \in \Sigma_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, moreover, for any $i \in A$, either $\sigma \notin \Sigma_{i,c}$, or $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} \neq \emptyset$, i.e., there is $s' \in \overline{K}$ and $u' \in \Sigma_{uc}^*$ such that $P_i(s) = P_i(s')$ and $s'u'\sigma \in \overline{K}$. Therefore, by Eq. (6), we have $\sigma \in S_{P_i}(P_i(s))$ for any $i \in A$. So $\sigma \in S_A(s)$ by Eq. (7). Notice that $su \in \overline{K} = L(G, S_A)$ and $su\sigma \in L(G)$, by the definition of $L(G, S_A)$, we obtain $su\sigma \in L(G, S_A) = \overline{K}$, which is in contradiction with $su\sigma \in L(G) - \overline{K}$.

($\Leftarrow$) For $s \in \Sigma^*$ and $i \in I$, define the local supervisors under communication delays $S_{P_i}(P_i(s))$ and A-decentralized supervisor S_A as Eq. (6) and Eq. (7), respectively. In order to prove $L(G, S_A) = \overline{K}$, i.e., for all $s \in \Sigma^*$,

$$s \in L(G, S_A) \quad \text{iff} \quad s \in \overline{K}, \tag{10}$$

we show it by induction on the length $|s|$.

If $|s| = 0$, i.e., $s = \epsilon$, the base case holds obviously. Suppose that (10) holds for any string s with $|s| \leq n$. The following is to prove it for $s\sigma$ where $|s| = n$ and $\sigma \in \Sigma$.

Let $s\sigma \in L(G, S_A)$. By the definition of $L(G, S_A)$ and induction hypothesis, $s \in \overline{K}$, $s\sigma \in L(G)$ and $\sigma \in S_A(s)$. We verify $s\sigma \in \overline{K}$ from the following two cases.

Case 1: If $\sigma \in \Sigma_{A,uc}$, then it is clear that $s\sigma \in \overline{K}$ for the $\Sigma_{A,uc}$-controllability of K.

Case 2: If $\sigma \in \Sigma_{A,c}$, then we show $s\sigma \in \overline{K}$ by contradiction. Suppose that $s\sigma \notin \overline{K}$. From $\sigma \in S_A(s)$ and Eqs. (6) (7), for any $i \in A$, either $\sigma \in (\Sigma_c - \Sigma_{i,c})$ or there is $s' \in \overline{K}$ and $u \in \Sigma_{uc}^*$ such that $P_i(s) = P_i(s')$ and $s'u\sigma \in \overline{K}$, i.e., $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} \neq \emptyset$, which contradicts the delay-$\Sigma_{A,c}$-coobservability of K.

Conversely, let $s\sigma \in \overline{K}$. By the induction hypothesis, $s \in L(G, S_A)$ for $s \in \overline{K}$. If $\sigma \in \Sigma_{A,uc}$, then $\sigma \in S_A(s)$ from Eq. (7). If $\sigma \in \Sigma_{A,uc}$, we check $\sigma \in S_A(s)$ by contradiction. Assume that $\sigma \notin S_A(s)$. According to Eqs. (6) (7), there is $i \in A$ such that $\sigma \in \Sigma_{i,c}$ and $s'u\sigma \notin \overline{K}$ for any $s' \in \overline{K}$ and any $u \in \Sigma_{uc}^*$ with $P_i(s) = P_i(s')$. As a result, for $s' = s$ and $u = \epsilon$, we obtain $s'u\sigma = s\sigma \notin \overline{K}$, which contradicts the assumption of $s\sigma \in \overline{K}$. Therefore, we also have $\sigma \in S_A(s)$. Consequently, $\sigma \in S_A(s)$ together with $s\sigma \in L(G)$ and $s \in L(G, S_A)$ implies $s\sigma \in L(G, S_A)$.

Definition 6: Let $A \in 2^I$. The A-decentralized supervisor S_A is called to be *nonblocking* if $\overline{L_m(G, S_A)} = L(G, S_A)$.

Theorem 2: Let $A \in 2^I$ and $K \subseteq L(G)$. There is a nonblocking A-decentralized supervisor under communication delays S_A such that $L(G, S_A) = \overline{K}$ and $L_m(G, S_A) = K$ if and only if K is $\Sigma_{A,uc}$-controllable, delay-$\Sigma_{A,c}$-coobservable and $L_m(G)$-closed.

Proof: ($\Rightarrow$) The $\Sigma_{A,uc}$-controllability and delay-$\Sigma_{A,c}$-coobservability of K have been proved in Theorem 1. From

$L(G, S_A) = \overline{K}$ and $L_m(G, S_A) = K$, we know that K is $L_m(G)$-closed since

$$K = L_m(G, S_A) = L(G, S_A) \cap L_m(G) = \overline{K} \cap L_m(G).$$

($\Leftarrow$) We define the local supervisors under communication delays and A-decentralized supervisor as Eqs. (6) (7). By Theorem 1, we have $L(G, S_A) = \overline{K}$. Furthermore, since K is $L_m(G)$-closed, we have

$$K = \overline{K} \cap L_m(G) = L(G, S_A) \cap L_m(G) = L_m(G, S_A).$$

As a result, $\overline{L_m(G, S_A)} = \overline{K} = L(G, S_A)$, that is, S_A is nonblocking.

IV. Reliable Decentralized Supervisory Control under Communication Delays

Based on the results presented in Section 3, we are ready to investigate the reliable decentralized supervisor under communication delays.

Definition 7: Let system G be jointly controlled by n local supervisors under communication delays $S_{P_1}, S_{P_2}, \cdots, S_{P_n}$ and $K \subseteq L(G)$. The decentralized supervisor under communication delays is said to be *k-reliable*, if $L(G, S_A) = \overline{K}$ for any $A \in 2^I$ with $|A| \geq k$, where $1 \leq k \leq n$, $|A|$ represents the number of elements of A.

Intuitively, a k-reliable decentralized supervisor under communication delays achieves exactly the desired specification under possible failure of any less than or equal to $n-k$ local supervisors with communication delays.

For $i \in I$, denote $\widetilde{\Sigma}_{i,uc} = \Sigma - \widetilde{\Sigma}_{i,c}$, where

$$\widetilde{\Sigma}_{i,c} = \{\sigma \in \Sigma_{i,c} : |In(\sigma)| \geq n - k + 1\}.$$

Let $A \in 2^I$, define $\widetilde{\Sigma}_{A,c} = \cup_{i \in A} \widetilde{\Sigma}_{i,c}$ and $\widetilde{\Sigma}_{A,uc} = \Sigma - \widetilde{\Sigma}_{A,c}$.

For the sake of simplicity, when $A = I$, we denote $\widetilde{\Sigma}_c = \widetilde{\Sigma}_{I,c}$, $\widetilde{\Sigma}_{uc} = \widetilde{\Sigma}_{I,uc}$, and $S_{dec} = S_I$.

Definition 8: A language $K \subseteq L(G)$ is said to be *$\widetilde{\Sigma}_{uc}$-controllable* if $\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) \subseteq \overline{K}$.

Definition 9: Let $1 \leq k \leq n$. A language $K \subseteq L(G)$ is said to be *k-reliably delay-$\widetilde{\Sigma}_c$-coobservable*, if for any $su \in \overline{K}$ and any $\sigma \in \widetilde{\Sigma}_c$ with $u \in \widetilde{\Sigma}_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, we have

$$|A_{s,u,\sigma}| \geq n - k + 1,$$

where

$$A_{s,u,\sigma} = \{i \in In(\sigma) : (P_i^{-1} P_i(s) \cap \overline{K}) \Sigma_{uc}^* \sigma \cap \overline{K} = \emptyset\}. \tag{11}$$

Remark 3: From Definition 5, we know that the delay-Σ_c-coobservability of K is equivalent to the k-reliably delay-$\widetilde{\Sigma}_c$-coobservability of K with $k = n$. In addition, Definition 9 generalizes the corresponding concept of [10] to the case under communication delays. If the terms of 'u' and 'Σ_{uc}^*' are removed, then Definition 9 degenerates to Definition 2 of [10] (page 662 of [10]).

Lemma 1: Let $1 \leq k \leq n$ and $K \subseteq L(G)$. There is a k-reliable decentralized supervisor under communication delays, if and only if, K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable for any $A \in I_k$, where $I_k = \{A \in 2^I : |A| = k\}$.

Proof: ($\Rightarrow$) Assume that there is a k-reliable decentralized supervisor, then $L(G, S_B) = \overline{K}$ for any $B \in 2^I$ with $|B| \geq k$. So $L(G, S_A) = \overline{K}$ for any $A \in I_k$. By Theorem 1, K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable.

($\Leftarrow$) Assume that K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable for any $A \in I_k$. We define the local supervisors under communication delays as follows: for $i \in I$,

$$\begin{aligned} S_{P_i}(P_i(s)) \;\; = \;\; & \{\sigma \in \Sigma_{i,c} : (\exists s^{'} \in \overline{K})(\exists u \in \Sigma_{uc}^*) \\ & [P_i(s) = P_i(s^{'}) \wedge s^{'}u\sigma \in \overline{K}]\} \\ & \cup (\Sigma_c - \Sigma_{i,c}) \cup \Sigma_{uc}. \end{aligned} \tag{12}$$

Next, we prove that the decentralized supervisor synthesized by the above local supervisors is k-reliable, that is, $L(G, S_A) = \overline{K}$ for any $A \in 2^I$ with $|A| \geq k$, where S_A is defined as

$$S_A(s) = (\bigcap_{i \in A} S_{P_i}(P_i(s))) \cup \Sigma_{A,uc}. \tag{13}$$

If $|A| = k$, then from the assumption, K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable. By Theorem 1, we have $L(G, S_A) = \overline{K}$. If $|A| > k$, then there are $B_1, B_2, \cdots, B_m$ such that $A = B_1 \cup B_2 \cup \cdots \cup B_m$ and $B_i \in I_k$ for each B_i. By the assumption, K is $\Sigma_{B_i,uc}$-controllable and delay-$\Sigma_{B_i,c}$-coobservable for each B_i. According to Theorem 1, we have

$$L(G, S_{B_1}) = L(G, S_{B_2}) = \cdots = L(G, S_{B_m}) = \overline{K},$$

which implies $L(G, S_A) = \overline{K}$ according to Proposition 1. So the decentralized supervisor is k-reliable.

Lemma 2: Let $1 \leq k \leq n$ and $K \subseteq L(G)$. K is $\widetilde{\Sigma}_{uc}$-controllable and k-reliably delay-$\widetilde{\Sigma}_c$-coobservable, if and only if, K is $\Sigma_{A,uc}$-controllable and delay-$\Sigma_{A,c}$-coobservable for any $A \in I_k$, where $I_k = \{A \in 2^I : |A| = k\}$.

Proof: ($\Leftarrow$) (1) We first prove that K is $\widetilde{\Sigma}_{uc}$-controllable, i.e., $\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) \subseteq \overline{K}$. Take an arbitrary $A \in I_k$, we have $\widetilde{\Sigma}_{uc} = \Sigma_{A,uc} \cup (\widetilde{\Sigma}_{uc} - \Sigma_{A,uc})$, and then

$$\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) = \overline{K}[\Sigma_{A,uc} \cup (\widetilde{\Sigma}_{uc} - \Sigma_{A,uc})] \cap L(G). \tag{14}$$

On the one hand, $\overline{K}\Sigma_{A,uc} \cap L(G) \subseteq \overline{K}$ because K is $\Sigma_{A,uc}$-controllable for any $A \in I_k$. On the other hand, for any $s \in \overline{K}$ and any $\sigma \in \widetilde{\Sigma}_{uc} - \Sigma_{A,uc}$ satisfying $s\sigma \in L(G)$, we have $\sigma \in \Sigma_c$ and $|In(\sigma)| \leq n - k$, which indicates that there is $A^{'} \in I_k$ such that $In(\sigma) \cap A^{'} = \emptyset$, i.e., $\sigma \in \Sigma_{A^{'},uc}$. Since K is $\Sigma_{A,uc}$-controllable for any $A \in I_k$, we obtain that $\overline{K}\Sigma_{A^{'},uc} \cap L(G) \subseteq \overline{K}$ for $A^{'} \in I_k$, and then $s\sigma \in \overline{K}$. So $\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) \subseteq \overline{K}$.

(2) Next, we verify that K is k-reliably delay-$\widetilde{\Sigma}_c$-coobservable by contradiction.

Suppose that K is not k-reliably delay-$\widetilde{\Sigma}_c$-coobservable. By Definition 9, there is $su \in \overline{K}$ and $\sigma \in \widetilde{\Sigma}_c$ satisfying $u \in \widetilde{\Sigma}_{uc}^*$, $su\sigma \in L(G) - \overline{K}$ and $|A_{s,u,\sigma}| \leq n - k$. Due to

$\sigma \in \widetilde{\Sigma}_c$, we obtain $|In(\sigma)| \geq n-k+1$. That is, there is $B \in I_k$ satisying $A_{s,u,\sigma} \cap B = \emptyset$ and $In(\sigma) \cap B \neq \emptyset$ (i.e., $\sigma \in \Sigma_{B,c}$). From the assumption, K is delay-$\Sigma_{B,c}$-coobservable for B since $B \in I_k$. Therefore, for the above su and σ, there exists $i \in B$ such that $i \in In(\sigma)$ and $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset$, i.e., $i \in A_{s,u,\sigma}$. Hence $i \in A_{s,u,\sigma} \cap B$, which is in contradiction with $A_{s,u,\sigma} \cap B = \emptyset$.

($\Rightarrow$) For any $A \in I_k$,

$$\begin{aligned} \Sigma_{A,uc} &= \Sigma_{uc} \cup \{\sigma \in \Sigma_c : \sigma \notin \Sigma_{A,c}\} \\ &\subseteq \Sigma_{uc} \cup \{\sigma \in \Sigma_c : |In(\sigma)| \leq n-k\} = \widetilde{\Sigma}_{uc}. \end{aligned} \tag{15}$$

From the $\widetilde{\Sigma}_{uc}$-controllability of K (i.e., $\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) \subseteq \overline{K}$), we have $\overline{K}\Sigma_{A,uc} \cap L(G) \subseteq \overline{K}$, that is, K is $\Sigma_{A,uc}$-controllable for $A \in I_k$.

Next, we prove that K is delay-$\Sigma_{A,c}$-coobservable for any $A \in I_k$. For any given $su \in \overline{K}$ and $\sigma \in \Sigma_{A,c}$ satisfying $u \in \Sigma_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, we have $\sigma \in \widetilde{\Sigma}_c$ (otherwise, by the $\widetilde{\Sigma}_{uc}$-controllability of K, $su\sigma \in \overline{K}$, which is in contradiction with $su\sigma \in L(G) - \overline{K}$). Therefore, $|In(\sigma)| \geq n-k+1$. Notice that K is k-reliably delay-$\widetilde{\Sigma}_c$-coobservable, we have $|A_{s,u,\sigma}| \geq n-k+1$. Due to $A \in I_k$ (i.e., $|A| = k$), it is obtained that $A \cap A_{s,u,\sigma} \neq \emptyset$, that is, there is $i \in A$ such that $\sigma \in \Sigma_{i,c}$ and $(P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset$.

From Lemma 1 and Lemma 2, we obtain the necessary and sufficient conditions for the existence of a k-reliable decentralized supervisor under communication delays.

Theorem 3: Let $1 \leq k \leq n$ and $K \subseteq L(G)$. There is a k-reliable decentralized supervisor under communication delays, if and only if, K is $\widetilde{\Sigma}_{uc}$-controllable and k-reliably delay-$\widetilde{\Sigma}_c$-coobservable.

Proof: It is a combination of Lemma 1 and Lemma 2.

Remark 4: The existence of a k-reliable decentralized supervisor without any delay was investigated in [10]. Theorem 3 presented above generalizes the results of [10] to the case under communication delays.

Theorem 4: Let $1 \leq k \leq n$ and $K \subseteq L(G)$. There is a nonblocking k-reliable decentralized supervisor under communication delays such that $L_m(G, S_A) = K$ for any $A \in 2^I$ with $|A| \geq k$, if and only if, K is $\widetilde{\Sigma}_{uc}$-controllable, k-reliably delay-$\widetilde{\Sigma}_c$-coobservable and $L_m(G)$-closed.

Proof: ($\Rightarrow$) Due to Theorem 3, we only need to prove that K is $L_m(G)$-closed. From Definition 3 and $L_m(G, S_A) = K$, it is clearly obtained that K is $L_m(G)$-closed since

$$K = L_m(G, S_A) = L(G, S_A) \cap L_m(G) = \overline{K} \cap L_m(G).$$

($\Leftarrow$) By Theorem 3, there is a k-reliable decentralized supervisor under communication delays, i.e., $L(G, S_A) = \overline{K}$ for any $A \in 2^I$ with $|A| \geq k$. Since K is $L_m(G)$-closed, we have $K = \overline{K} \cap L_m(G) = L(G, S_A) \cap L_m(G) = L_m(G, S_A)$. Therefore, $K = L_m(G, S_{dec})$ and $\overline{L_m(G, S_{dec})} = \overline{K} = L(G, S_{dec})$, that is, the k-reliable decentralized supervisor under communication delays is nonblocking.

Remark 5: When $k = n$, the conditions of Theorem 4 are reduced to the existence conditions of the nonblocking decentralized supervisor presented in [8]. So Theorem 4 generalizes the result of [8] on the existence of a nonblocking decentralized supervisor under communication delays.

V. Illustrative Examples

According to Theorem 3, we know that the existence of a k-reliable decentralized supervisor under communication delays can be checked by the $\widetilde{\Sigma}_{uc}$-controllability and k-reliably delay-$\widetilde{\Sigma}_c$-coobservability of K.

Example 2: Consider a DES G modeled by an automaton shown in Fig. 3. Let $n = 3$ (i.e., $I = \{1, 2, 3\}$) and

$$\begin{aligned} &\Sigma_{1,o} = \{\sigma_1, \sigma_2, \sigma_5\}, \quad \Sigma_{2,o} = \{\sigma_1, \sigma_4\}, \quad \Sigma_{3,o} = \{\sigma_2, \sigma_3\}; \\ &\Sigma_{1,c} = \{\sigma_1, \sigma_2\}; \qquad \Sigma_{2,c} = \{\sigma_1, \sigma_4\}, \quad \Sigma_{3,c} = \{\sigma_2, \sigma_3\}. \end{aligned}$$

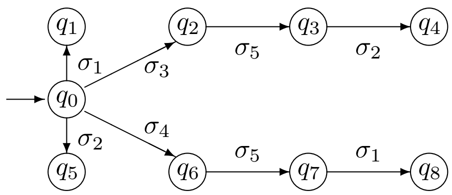

Fig. 3. A DES G.

Consider language

$$K = \overline{\sigma_3\sigma_5 + \sigma_4\sigma_5},$$

then $K \subseteq L(G)$.

In the following, we verify that there is a 2-reliable decentralized supervisor under communication delays by testing the $\widetilde{\Sigma}_{uc}$-controllability and k-reliably delay-$\widetilde{\Sigma}_c$-coobservability of K, where

$$\widetilde{\Sigma}_c = \{\sigma \in \Sigma_c : |In(\sigma)| \geq 2\} = \{\sigma_1, \sigma_2\},$$

and $\widetilde{\Sigma}_{uc} = \Sigma - \widetilde{\Sigma}_c = \{\sigma_3, \sigma_4, \sigma_5\}$.

(1) K is $\widetilde{\Sigma}_{uc}$-controllable because

$$\overline{K}\widetilde{\Sigma}_{uc} \cap L(G) = \{\sigma_3, \sigma_4, \sigma_3\sigma_5, \sigma_4\sigma_5\} \subseteq \overline{K}.$$

(2) Next, we prove that K is 2-reliably delay-$\widetilde{\Sigma}_c$-coobservable, i.e., for any $su \in \overline{K}$ and any $\sigma \in \widetilde{\Sigma}_c$ with $u \in \widetilde{\Sigma}_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, the following holds

$$|A_{s,u,\sigma}| \geq n - k + 1 = 3 - 2 + 1 = 2,$$

where

$$A_{s,u,\sigma} = \{i \in In(\sigma) : (P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset\}. \tag{16}$$

We list all cases of s, u and σ satisfying $su \in \overline{K}$, $\sigma \in \widetilde{\Sigma}_c$, $u \in \widetilde{\Sigma}_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, and check the 2-reliably delay-$\widetilde{\Sigma}_c$-coobservability of K for all cases in Table I, in which for the sake of simplicity, we denote

$$I(s, \sigma) = \{i \in I : (P_i^{-1}P_i(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K} = \emptyset\},$$

and $A_{s,u,\sigma} = In(\sigma) \cap I(s, \sigma)$.

Note that all elements in the rightmost column of Table I are "True", that is, $|A_{s,u,\sigma}| \geq 2$ holds for all cases. Therefore, K is 2-reliably delay-$\widetilde{\Sigma}_c$-coobservable.

According to Theorem 3, there is a 2-reliable decentralized supervisor under communication delays.

Table I. Testing the 2-reliably delay-$\widetilde{\Sigma}_c$-coobservability.

σ	s	u	$I(s,\sigma)$	$A_{s,u,\sigma}$	$\lvert A_{s,u,\sigma}\rvert \geq 2$
σ_1	ϵ	ϵ	$\{1,2,3\}$	$\{1,2\}$	True
	ϵ	$\sigma_4\sigma_5$	$\{1,2,3\}$	$\{1,2\}$	True
	σ_4	σ_5	$\{1,2,3\}$	$\{1,2\}$	True
	$\sigma_4\sigma_5$	ϵ	$\{1,2,3\}$	$\{1,2\}$	True
σ_2	ϵ	ϵ	$\{1,2,3\}$	$\{1,3\}$	True
	ϵ	$\sigma_3\sigma_5$	$\{1,2,3\}$	$\{1,3\}$	True
	σ_3	σ_5	$\{1,2,3\}$	$\{1,3\}$	True
	$\sigma_3\sigma_5$	ϵ	$\{1,2,3\}$	$\{1,3\}$	True

In fact, from Eqs. (6) (7), the local supervisors under communication delays can be designed as follows:

$$S_{P_1}(P_1(s)) = \{\sigma_3,\sigma_4,\sigma_5\},$$
$$S_{P_2}(P_2(s)) = \begin{cases} \{\sigma_2,\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, \\ \{\sigma_2,\sigma_3,\sigma_5\}, & \text{otherwise.} \end{cases}$$
$$S_{P_3}(P_3(s)) = \begin{cases} \{\sigma_1,\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_3(s)=\epsilon, \\ \{\sigma_1,\sigma_4,\sigma_5\}, & \text{otherwise.} \end{cases}$$

For any $A \in 2^{\{1,2,3\}}$ with $|A| \geq 2$, the A-decentralized supervisors under communication delays S_A are synthesized as follows:

$$S_{\{1,2\}}(s) = \begin{cases} \{\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, \\ \{\sigma_3,\sigma_5\}, & \text{otherwise.} \end{cases}$$
$$S_{\{1,3\}}(s) = \begin{cases} \{\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_3(s)=\epsilon, \\ \{\sigma_4,\sigma_5\}, & \text{otherwise.} \end{cases}$$
$$S_{\{2,3\}}(s) = \begin{cases} \{\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, P_3(s)=\epsilon, \\ \{\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, P_3(s)\neq\epsilon, \\ \{\sigma_3,\sigma_5\}, & \text{if} \quad P_2(s)\neq\epsilon, P_3(s)=\epsilon, \\ \{\sigma_5\}, & \text{if} \quad P_2(s)\neq\epsilon, P_3(s)\neq\epsilon. \end{cases}$$
$$S_{\{1,2,3\}}(s) = \begin{cases} \{\sigma_3,\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, P_3(s)=\epsilon, \\ \{\sigma_4,\sigma_5\}, & \text{if} \quad P_2(s)=\epsilon, P_3(s)\neq\epsilon, \\ \{\sigma_3,\sigma_5\}, & \text{if} \quad P_2(s)\neq\epsilon, P_3(s)=\epsilon, \\ \{\sigma_5\}, & \text{if} \quad P_2(s)\neq\epsilon, P_3(s)\neq\epsilon. \end{cases}$$

By Definition 3, we can calculate that

$$\begin{aligned} & L(G,S_{\{1,2\}}) = L(G,S_{\{1,3\}}) = L(G,S_{\{2,3\}}) \\ = \; & L(G,S_{\{1,2,3\}}) = \{\epsilon,\sigma_3,\sigma_4,\sigma_3\sigma_5,\sigma_4\sigma_5\} = \overline{K}. \end{aligned}$$

Therefore, by Definition 7, we know that the decentralized supervisor under communication delays is 2-reliable.

Example 3: Consider the DES G shown as that in Example 2, but the sublanguage is changed by

$$K = \overline{\sigma_3\sigma_5\sigma_2 + \sigma_4\sigma_5}.$$

In the following, we verify that K is not 2-reliably delay-$\widetilde{\Sigma}_c$-coobservable, where $\widetilde{\Sigma}_c = \{\sigma_1,\sigma_2\}$.

Take $s=\epsilon$, $u=\epsilon$ and $\sigma=\sigma_2$, then $su \in \overline{K}$, $\sigma \in \widetilde{\Sigma}_c$, $u \in \widetilde{\Sigma}_{uc}^*$ and $su\sigma \in L(G) - \overline{K}$, but $i = 1 \notin A_{s,u,\sigma}$ since

$$\sigma_3\sigma_5\sigma_2 \in (P_1^{-1}P_1(s) \cap \overline{K})\Sigma_{uc}^*\sigma \cap \overline{K},$$

and $i=2 \notin A_{s,u,\sigma}$ for $\sigma \notin \Sigma_{2,c}$. So $|A_{s,u,\sigma}| \leq 1$. That is to say, K is not 2-reliably delay-$\widetilde{\Sigma}_c$-coobservable.

According to Theorem 3, there is not a 2-reliable decentralized supervisor under communication delays.

VI. Conclusion

In this paper, the problem of reliable decentralized supervisory control of DESs with communication delays was investigated in the framework of [8], and the results of [10], [11] were generalized. The notations of $\widetilde{\Sigma}_{uc}$-controllability and k-reliably delay-$\widetilde{\Sigma}_c$-coobservability of a sublanguage were formulated, based on which some necessary and sufficient conditions for the existence of a k-reliable decentralized supervisor under communication delays were proposed. These results can be reduced to those in [10] if the communication delays are negligible. Moreover, the results of [8] can be regarded as a special case of the proposed k-reliable decentralized control with $k=n$. With the results obtained in this paper, we will consider the reliable robust nonblocking supervisory control of DESs with communication delays and investigate the reliable supervisory control of stochastic DESs based on our previous work of [4] [5] in the subsequent work.

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