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Weights' Assignment in Multi-Agent Systems Under a Time-Varying Topology

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Abstract— **The main objective of this paper is to design an optimal solution for weights' assignment in formation and reconfiguration control among a group of robots. The optimal solution must keep the control effort given to the whole system at its minimum possible level and guarantees that the desired configuration can be reached. In particular, the case of a single leader under a time-varying topology is considered. In contrast to the existing literature on this topic, we assume that the graph is weighted and time-varying; moreover, weights can be assigned freely. Under this setup, there are plenty of possible weights. However, determining a set of the best weights remains as an open problem because the control effort given to the agents must be minimized meanwhile the final desired states must be assured. In this paper, the problem of weights' assignment is discussed and formulated using the optimal control theory. The optimal control strategy is designed based on minimization of an index function and a solution is found using Hamiltton-Jacobi-Bellman equations. Finally, some simulation results are presented to illustrate the approach.**

I. Introduction

Recent developments in communication and computation systems have made a revolution in the area of multi-agent systems, which have seen a wide range of applications in health monitoring, underwater or space exploration, intelligent transportation systems and rapid emergency response [20]; moreover, they have many military applications such as target acquisition, reconnaissance missions where environments involved with numerous uncertainties [3]. The key feature of multi-agent systems is that the group behavior of multiple agents is not simply a summation of the individual agent's behavior. Although each individual agent's dynamics and their interaction rules could be very simple, a large collection of these elementary agents, as a whole, could exhibit remarkable capabilities and display highly complex behaviors. Hence, a cooperative control of such a complicated system becomes really challenging.

Cooperative control of multi-agent systems is still in its infancy stages and poses significant theoretical and technical challenges [23]- [13]. The cooperative control of such complex networked systems have highly inspired by their biological counterparts such as fish schooling, bees flocking and ant colonies. See for instance, [4], [2], [12], [16] and references therein. Recently, different approaches like graph Laplacian for the associated neighborhood graphs, artificial potential functions, and navigation functions for distributed formation stabilization with collision avoidance constraints have been developed in this field.

Early efforts have been found on consensus seeking, convergence to a common value, among distributed agents. During the last years, different approaches such as graph Laplacian for the associated neighborhood graphs and artificial potential functions have been developed in this area. For instance, [22] solved the consensus problem stable flocking of the mobile agents and [18] applied linear and nonlinear consensus protocols to a group of networked agents with integrator dynamics. Also, interested readers can refer to [15], [9] and their references. Although consensus law application in networked system is recent, it used to be focus of attention for long time by computer scientists [15]. Much work has been done on the formation stabilization and consensus seeking, see e.g., [15], [17], [9], [22]. Moreover, the stability problem for a group of swarm was discussed in [7]. Authors in [19] solved the coordination control problem for multi-agent systems with help of appropriate Lyapunov function candidates.

Consensus is more considered about stability problem, but although it is important to keep rigid formation, it could be more important to make multi-agent systems reconfigurable between different formations. The main question is that whether agents can be steered to any desired configuration or not. Align the same line, [21] modified the consensus law's structure and formulated it as a controllability problem; henceforth, some sufficient and necessary algebraic conditions for this problem was introduced. This problem is further discussed in [10] and [11]. In contrast to the existing literature on this topic, we [24] assumed the graph to be weighted and we might freely assign the weights. Under this setup, the system is controllable if one may find a set of weights so as to satisfy the classical controllability rank condition. Moreover, a novel notion of multi-agent structural controllability was proposed; it turned out that this controllability notation, purely depends on the topology of the communication scheme, and the multi-agent system is controllable if and only if the graph is connected.

Following [24], there are plenty of weight sets which can drive the system to its desired destination; however, not all of them are able to give the best performance in the sense that they can keep the control effort given to the system at its minimum possible level. Although the Linear Quadratic Gaussian controller was proposed to minimize the the leader's control effort [25], yet there is a need for a more general solution which modifies the control efforts given to the whole group. The solution must consider both plant

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limitations and final conditions. Hence, global optimization for a group of robots remains as an open problem.

In contrast to the existing literature, we consider the topology to be both time-varying and weighted. In addition, it is assumed that there exist only one leader in the group' the whole system dynamics is aggregated and the global solution is proposed. Moreover, the optimal solution for weights' assignment is developed such that it considers all individuals and significance of final goal reaching.

The rest of the paper is structured as follows. In the next section, some of the necessary notations are introduced and the problem studied in this paper is formulated. In Section III, the problem of weights' assignment is written as optimal control problem and HJB equations are used to solve the cost function. Section IV presents some numerical examples to illustrate the derived theoretical results and design methods. Finally, the paper concludes with comments and plans for further work.

II. Notations and problem formulation

A. Notation

This section provides some graph theoretic objects and their properties that are going to be used in sequel sections. The [8] has provided more details.

A weighted graph is an appropriate representation for the communication or sensing links among agents because it can represent both existence and strength of these links among agents. The weighted graph $\mathcal{G}$ with N vertices consists of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and an edge set $\mathcal{I} = \{e_1, e_2, \ldots, e_N\}$, which is the interconnection links among the vertices. Each edge in the weighted graph represents a bidirectional communication or sensing media. The *order* of the weighted graph is denoted to be the cardinality of its vertex set. Similarly, the cardinality of the edge set is defined as its *degree*. Two vertices are known to be *neighbors* if $(i, j) \in e$, and the number of neighbors for each vertex is its *valency*. An alternating sequence of distinct vertices and edges in the weighted graph is called a *path*. The weighted graph is said to be *connected* if there exists at least one path between any distinct vertices, and *complete* if all vertices are neighbors to each other.

The *adjacency matrix*, $\mathcal{A}$, is defined as

$$\mathcal{A}_{(i,j)} = \begin{cases} w_{ij}(t) & (i,j) \in e, \\ 0 & \text{otherwise}, \end{cases}$$

where $w_{ij} \neq 0$ stands for the weight of edge (i, j). Here, the adjacency matrix $\mathcal{A}$ is $|\mathcal{V}| \times |\mathcal{V}|$ and $|.|$ is the cardinality of a set.

Define another $|\mathcal{V}| \times |\mathcal{V}|$ matrix, $\mathcal{D}$, called *degree matrix*, as a diagonal matrix which consists of the degree numbers of all vertices.

The *Laplacian matrix* of a graph $\mathcal{G}$, denoted as $\mathcal{L}(\mathcal{G}) \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$ or $\mathcal{L}$ for simplicity, is defined as

$$\mathcal{L}_{(i,j)} = \begin{cases} \sum_{i\neq j} w_{ij}(t) & i = j, \\ -w_{ij}(t) & (i,j) \in e, \\ 0 & \text{otherwise}. \end{cases}$$

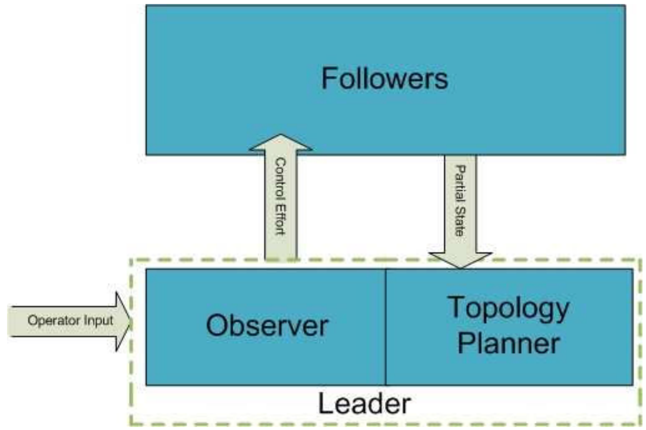

Fig. 1. The leader based observer

B. Problem formulation

In this paper, we consider the topology to be time-varying and weighted . We assume that there exist an agent which serves as the leader and the rest of agents are followers and take controls from the nearest neighbor law.

Consider N point mass agents with first order dynamics

$$\dot{x}_i = u_i, \qquad i = 1, ..., N, \tag{1}$$

where x_i is denoted to be the state of each agent and can have arbitrary dimension but all agents are required to have the same dimension. Even though the analysis that follows remains valid any dimension n, for the sake of simplicity we will present the one-dimensional case. All expressions can be easily generalized to higher dimensions case via Kronecker product.

Without loss of generality, assume that the N-th agent serves as the leader and takes commands and controls from outside operators directly,

$$\dot{x}_N = u_N. \tag{2}$$

While the rest $N-1$ agents are followers and take controls as the nearest neighbor law :

$$u_i = -\sum\nolimits_{j\in\mathcal{N}_i} w_{ij}(t)(x_i - x_j), \tag{3}$$

where $\mathcal{N}_i$ is the neighbor set of the agent i, $w_{ij} \in \mathbb{R}^+$ is weight of the edge from agent i to agent j. Moreover, not all of followers are able to communicate with the leader directly and the leader can establish communication link just to some of them. This problem can be formulated as

$$y_i = \lambda_{iN} w_{iN}(t) x_i, \tag{4}$$

where w_{iN} is weight of edge from agent i to the leader and λ_{iN} is defined as

$$\lambda_{iN} = \begin{cases} 1 & i \in \mathcal{N}_N, \\ 0 & \text{otherwise}. \end{cases}$$

The leader can just have an access to y_i and needs to observe the unknown states for designing of an appropriate control law; also, it requires the topology map for design of convenient control law. An operator provides the control effort needed for the leader. This setup is clearly shown in Fig. 1.

The algebraic graph theory [8] helps us to we rewrite the system dynamics (1), (2), (3), (4) into the following matrix form :

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{aq} & B_{aq} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ u_N \end{bmatrix} . \quad (5)$$

$$y = C_{aq} x$$

Or, equivalently

$$\begin{cases} \dot{x} & = & A_{aq}x + B_{aq}x_N \\ \dot{x}_N & = & u_N \\ y & = & C_{aq}x \end{cases} , \quad (6)$$

where $A_{aq} \in \mathbb{R}^{(N-1)\times(N-1)}$ and $B_{aq} \in \mathbb{R}^{(N-1)\times 1}$ are both sub-matrices of the corresponding graph Laplacian matrix $\mathcal{L}$. The matrix A_{aq} reflects the interconnection among followers, and the column vector B_{aq}, C_{aq} represents the relation between followers and the leader.

Our objective is to design a paradigm such that the control effort given to the whole group can be minimized. In next the section the optimal control approach is used for solving this problem.

III. Main result

In this section we assume that the communication topology remains connected during the whole maneuver. This assumption guarantees that the system is both observable and controllable and the solution exist.

A. Cost function definition

Let Σ be a linear time-invariant version of the system in (6).

Definition 1: The linear time-invariant system Σ is said to be structurally controllable if and only if there exists a set of fixed w_{ij} which can make the system Σ controllable.

Definition 2: The linear time-invariant system Σ is said to be structurally observable if and only if there exists a set of fixed w_{ij} which can make the system Σ observable.

Lemma 1: The multi-agent system Σ under the fixed communication topology $\mathcal{G}'$ is structurally controllable *if and only if* $\mathcal{G}'$ is connected.

Proof: See the proof in [24] ■

Lemma 2: The multi-agent system Σ under the communication fixed topology $\mathcal{G}'$ is structurally observable *if and only if* $\mathcal{G}'$ is connected.

Proof: See the proof in [25] ■

The next step is to design a global optimal control strategy which can put into account whole dynamics. Moreover, it should be able to minimize the control effort given to each agent.

Let us define the following index function for overall system:

$$J = \int_0^T \left[(A_{aq}x)^T \mathcal{Q}(A_{aq}x) + x^T \mathcal{S} x + u_N R u_N \right] dt + \\ (x(T) - x_f)^T E\,(x(T) - x_f)\,, \quad (7)$$

where x_f stands for the desired final position at the final time T, and $\mathcal{Q} > 0$, $\mathcal{S} > 0$ and $R > 0$ are specification matrices.

Remark 1: The cost function introduced in (7) is in a quadratic form. It is chosen such that it minimizes the control effort given to the whole system. It not only minimizes the leader's control effort, but also penalizes followers' control signals.

B. Hamiltton-Jacobi-Bellman(HJB) equations

The problem of finding a minimum value for general cost function, can be solved by help of HJB set of equations. This method is applicable to the general finite horizon case [6]. Assume a system with the following dynamics

$$\dot{X} = f(t, X, u), \quad (8)$$

with the following cost function to be minimized as

$$J = \int_0^T g(t, X, u)dt + \lambda(X(T)). \quad (9)$$

A set of HJB equations can be used to solve the optimal problem in (8) and (9) as follows :

$$-\frac{\partial W}{\partial t}(t, X) = \min_{u\in U} \Xi(t, X, u), \\ W(T, X) = \lambda(X(T)), \\ u^* = arg \min_{u\in U} \{\Xi(t, X, u)\}\,,$$

where W is so called value function.

$$\Xi(t, X, u) = g(t, X, u) + \frac{\partial W}{\partial X}(t, X) f(t, X, u).$$

The solvability of the above minimization problem is depend on whether the PDE can be solved or not. In another word, one needs to find the value function W such that it satisfies the PDE.

C. Optimal control problem for multi-agent systems

The HJB equations can be rewritten for the system given in (6):

$$-\frac{\partial W}{\partial t}(t, X) = \min_{u\in U} \Xi(t, X, u) \\ W(T, X) = x(T)^T E x(T) + \phi(T) \quad (10) \\ \Xi(t, X, u) = (A_{aq}x)^T \mathcal{Q}(A_{aq}x) + x^T \mathcal{S} x + u_N R u_N + \\ \frac{\partial W}{\partial x}(A_{aq}x + B_{aq}u). \quad (11)$$

Existence of a solution: The existence of solution to the above minimization problem can be guaranteed if certain controllability and observability conditions are satisfied [14]. Moreover, it was just shown that as long as the topology graph $\mathcal{G}'$ remains connected, the controllability and observability requirement are both realized. Thus, the existence of solution for this minimization problem is guaranteed.

The above minimization problem has the optimal control law in the form of :

$$u^* = -\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T, \tag{12}$$

and on of the possible choice for W can be expressed as

$$W = -\frac{1}{2}x^T K(t)x + \phi(t). \tag{13}$$

The following Lemma shows how parameter K can be calculated such that the PDEs in (10) and (11) have solutions.

Theorem 1: Assume a group of agent with dynamics (1) who are connected with the nearest neighbor law (3). The following control law would minimize the cost function (7).

$$u^* = -\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T, \tag{14}$$

where $K(t)$ satisfies the following equation:

$$-\dot{K} = 2(\mathcal{S}+A_{aq}^T\mathcal{Q}A_{aq}) + \frac{K^TR^{-1}K}{2} \quad K(T) = 2E. \tag{15}$$

Proof: Equation (11) can be written as:

$$\begin{aligned}\Xi(t,X,u^*) &= (A_{aq}x)^T\mathcal{Q}(A_{aq}x) + x^T\mathcal{S}x+ \\ &u_N^*Ru_N^* + \frac{\partial W}{\partial x}(A_{aq}x + B_{aq}u) \\ &= -\frac{\partial W}{\partial t}(t,x).\end{aligned} \tag{16}$$

By substituting W and u_N^* according to (13) and (14), accordingly, one gets

$$\begin{aligned}-x^T\frac{\dot{K}}{2}x - \dot{\phi} &= (A_{aq}x)^T\mathcal{Q}(A_{aq}x) + x^T\mathcal{S}x \\ &(-\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T)^TR(-\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T) + A_{aq},\end{aligned} \tag{17}$$

which can be written into a compact form

$$\begin{aligned}-x^T\frac{\dot{K}}{2}x - \dot{\phi} &= (A_{aq}x)^T\mathcal{Q}(A_{aq}x) + x^T\mathcal{S}x \\ &\frac{x^TK^TR^{-1}Kx}{4} + A_{aq}.\end{aligned} \tag{18}$$

By comparing the corresponding terms in x^Tx, we get

$$-\dot{K} = 2(\mathcal{S} + A_{aq}^T\mathcal{Q}A_{aq}) + \frac{K^TR^{-1}K}{2}. \tag{19}$$

On the other hand, final condition can be verified as follows

$$\begin{aligned}W(T,x(T)) &= \frac{1}{2}x(T)^TK(T)x(T) + \phi(T) \\ &= x(T)^TEx(T) + x_f^TEx_f - 2Ex_f^Tx(T).\end{aligned} \tag{20}$$

Again by comparing corresponding term in $x(T)^Tx(T)$,

$$K(T) = 2E. \tag{21}$$

This completes the proof. ■

Remark 2: The problem of weights' assignment can be solved by help of Theorem 1. The optimal law (14) can be replaced in (6); hence, entries of matrix A_{aq} are updated, or in another word weights among the agents are modified such that not only the leader's control effort become optimum, but also the control effort given to the whole system is optimized. The result in Remark 2 is further illustrated in the next section.

IV. Numerical Example

In this section, we give a numerical examples to illustrate the theoretical results demonstrated in the earlier sections. Assume topology as shown in Fig. 2 which consists of three followers and a leader. We assume that x represents each agent's position. The dynamics of whole system can be written as the following:

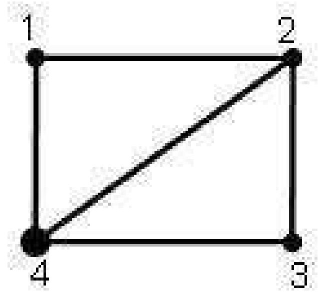

Fig. 2. A system consists of four agent and agent four serves as the leader

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0.25 & -0.25 & 0\\ -0.25 & 0.5 & -0.25\\ 0 & -0.25 & 0.25\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}-1\\ -1\\ -1\end{bmatrix}u. \tag{22}$$

Design parameters are set as below:

$$\mathcal{Q} = \begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix},\ E = \begin{bmatrix}0.1 & & \\ & 0.1 & \\ & & 0.1\end{bmatrix},$$

$$R = 1,\ \mathcal{S} = \begin{bmatrix}0.875 & 0.1875 & -0.0625\\ 0.1875 & 0.6250 & 0.1875\\ -0.0625 & 0.1875 & 0.8750\end{bmatrix}$$

One can write (15) for above setup as

$$-\dot{K} = 2I + \frac{K^2}{2}I \quad K(T) = 20I, \tag{23}$$

where I is the identity matrix. Above problem can be easily solved as

$$K = -2I \tan(2t - c),$$

where c can be obtained form the boundary condition. Henceforth, the feedback law can be written as

$$W = -\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)k + \phi(t)$$
$$\frac{\partial W}{\partial x} = -\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} k$$
$$u^* = -\frac{1}{2}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} k, \quad (24)$$

Where k is a diagonal element of the matrix K.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0.25 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} k.$$

Consequently, we get

$$\dot{X} = \begin{bmatrix} 0.25 + 0.5k & -0.25 + 0.5k & 0.5k \\ -0.25 + 0.5k & 0.5 + 0.5k & -0.25 + 0.5k \\ 0.5k & -0.25 + 0.5k & 0.25 + 0.5k \end{bmatrix} X,$$

where $X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. The above equation illustrates how the optimal solution in Theorem 1 can assign weights among a group of connected agents. The system (22) initiates form random initial conditions in $2D$ space and under the control law (24), all the followers are forced to converge into the origin within a finite time $t = 2$.

The system in (22) is exposed to the optimal control law (24). The states trajectory is depicted in Fig. 4. It is clearly shown in Fig. 5 that how followers are moving in 2D space till they reach the desired point. In Fig. 5 initial positions are marked by plus sign and the destination is illustrated by star sign. It can be seen from Fig. 5 that the proposed control strategy is capable of driving the system into its desired position. Furthermore, the control effort given to the system is shown in Fig. 3. Investigating Figures 3 and 5 reveals that not only the desired formation is obtained, but also control efforts given to the system is quite negligible. This supports that the optimal law in (24) has modified the weights such that control effort given to the system is optimized.

V. Conclusion

In this paper, the optimal control paradigm was proposed for weights' assignment among multi-agent systems under time-varying topology. It is assumed that system is under a leader. The problem of weights' assignment was written as an optimal control problem; henceforth, index function was minimized with the help of HJB equations. Finally, simulation results were introduced which underscore their theoretical counterparts. Our future work includes modifying

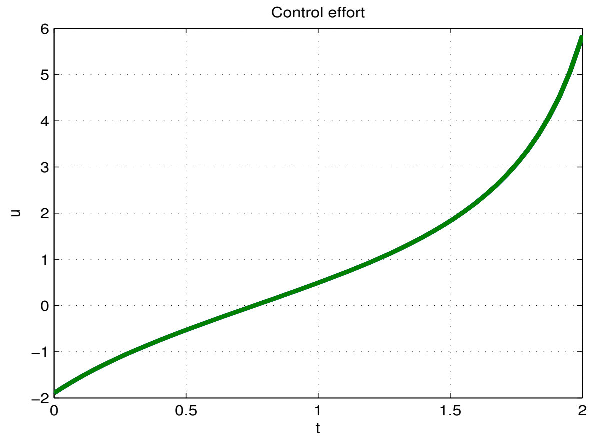

Fig. 3. The optimal control effort (24) given to the system (22).

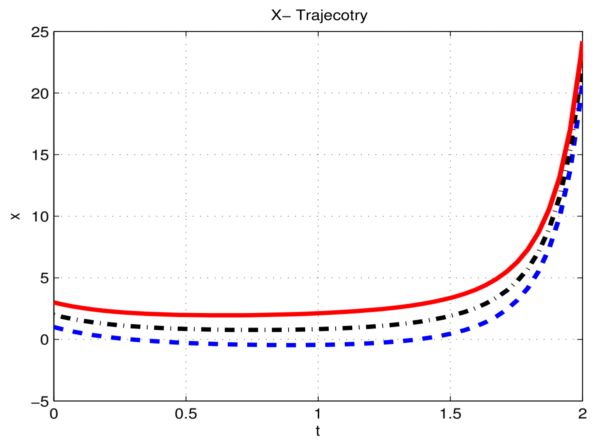

Fig. 4. X position trajectory of the system (22) which is driven by the optimal law (24) and design parameters (IV).

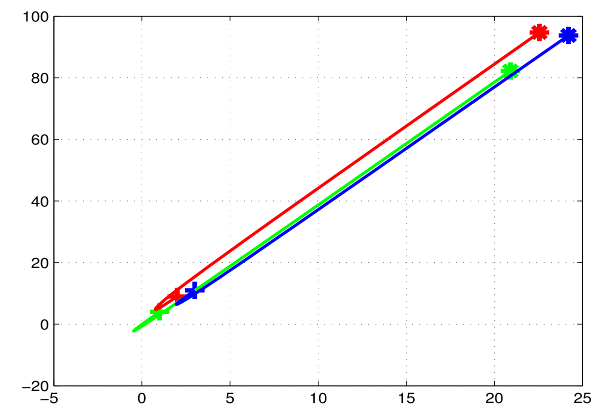

Fig. 5. The X-Y position trajectory of the system (22). Followers' initial positions (plus), final positions (star).

agents dynamics, optimizing the communication topology and optimizing the problem with respect to both the weights and the communication topology.

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