Scheduling-and-Control Codesign for a Collection of Networked Control Systems With Uncertain Delays

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***Abstract*—This paper is concerned with the simultaneous stabilization of a collection of continuous-time linear time-invariant (LTI) plants whose feedback-control loops are closed via a shared digital communication network. Because of the limitation of communication capacity, only a limited number of controller-plant connections can be accommodated at any time instant. Therefore, it is necessary to carefully determine a scheduling policy so as to achieve a simultaneous stabilization for all these control loops. A sufficient condition on the existence of such a scheduling policy is presented for a collection of networked LTI systems with sampled-data controllers and uncertain network-induced delays. The proof for the schedulability condition is in a constructive way, which can also serve as a systematic method to design a scheduling policy. Finally, a scheduling-and-feedback-control codesign procedure is proposed for the simultaneous stabilization of the collection of networked LTI systems, and the effectiveness of the proposed codesign procedure is demonstrated with simulation results.**

***Index Terms*—Average dwell time, codesign, networked control systems (NCSs), scheduling, switched systems.**

I. Introduction

NETWORKED control systems (NCSs) are feedback-control systems in which the communication between spatially distributed system components like sensors, actuators, and controllers occurs through shared band-limited digital communication networks. Compared with conventional point-to-point interconnected control systems, NCSs possess many attractive features due to the inclusion of a communication network. These advantages include higher system testability and resource utilization, as well as lower cost, reduced weight and power, and simpler installation and maintenance [1], [2], which make the use of networks in control systems connecting sensors/actuators to controllers more and more popular in many applications, including traffic control, satellite clusters, and mobile robots [3]–[5]. Consequently, considerable attention has been paid to the study of NCSs recently, see for example, the survey papers [4], [6], and [7], the recent special issues [8] and [9], and the references therein.

In NCSs, it is common that many spatially distributed system components, like sensors, controllers, and actuators, share a common communication network. This is very typical when using the base station to control and coordinate a group of mobile robots through a wireless network. Many specific application setups can be found in the real-world control systems through networks, such as controller area networks (CANs) [10] and Fieldbus [11]. For example, cart–pendulums are distributively controlled over CAN, which is taken from [10], as shown in Fig. 1. In this paper, we consider a class of NCSs consisting of a collection of continuous-time linear time-invariant (LTI) plants whose open-loop dynamics might be unstable. Each plant communicates with its remotely located controller over a shared network link, as shown in Fig. 2. Because of limited communication capacities, it is assumed that controller–plant communication is restricted, and not all the control loops in the NCSs can be addressed at the same time. That is, only a few controller–plant connections can be granted at any one time, while the other feedback-control loops are assumed to be open loop. It is clear that, if some connections monopolize the network, the other plants will not be stabilizable. In order to guarantee the stabilization of each plant, it is necessary to design a scheduling algorithm to schedule the NCSs.

It should be pointed out that similar problems have been studied in [12]–[16]. In [12], the rate-monotonic scheduling algorithm was applied to schedule a set of NCSs. A schedulability condition was presented in [13]–[15] for the simultaneous stability of a group of continuous-time linear systems by a common Lyapunov function, and a time-division-based scheduling policy was developed in [16] by employing the average dwell-time technique [17], [18] incorporated with piecewise Lyapunov-like functions. So far, these studies are carried out in the continuous-time domain and under the assumption that no time delay happens.

The network-induced delay is one of the basic problems in NCSs [1], and the characteristics of delay can be constant, bounded, or even random, which can usually degrade a system's performance and even cause system instability. For instance, a typical motion-control example was presented in [19], in which the time variation of the network-induced delays results

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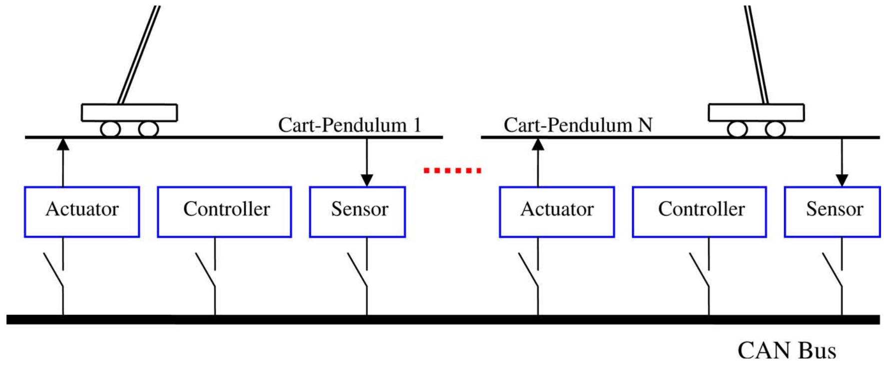

Fig. 1. Cart–pendulums are distributively controlled over CAN.

in an unstable system. In view of discretizing continuous-time systems affected by time-varying delays [19]–[23], these systems can be described by uncertain systems for whose robust control methods can be applied. Additionally, other effective approaches, such as the maximum allowable delay-bound approach combined with Lyapunov–Krasovskii functionals [22]–[24], the stochastic system approach [25], [26], the predictive control approach [27], [28], and the switched system approach [29]–[32] have been developed to study the modeling, analysis, and synthesis of NCSs with constant, random, or time-varying delays.

On the other hand, the problem of feedback-control-and-scheduling codesign has recently received increasing attention in the NCS literature, since NCS performances not only depend on the design of control algorithms but also on the scheduling of the shared network resources [12]. In [33], this problem was addressed to improve control performances for a linear system over limited bandwidth networks. A codesign scheme was presented in [34] for stabilizing an LTI system, in which only limited sensors/actuators can exchange information with a remote controller via a shared common communication network. In [35], a predictive control-and-scheduling-codesign approach was proposed to deal with the communication constraints for a set of NCSs with network-induced delays.

In this paper, we address the problem of feedback-control-and-scheduling codesign in a sampled-data control framework and explicitly consider uncertain network-induced delays in the control loops. The motivation comes from the popularity of digital control and unavoidable transmission delays in communication networks. The main contributions of this paper are that: 1) a constructive proof method is used to show that the schedulability condition only depends on the convergence rate of the closed-loop system and the divergence rate of the open-loop plant; 2) simultaneous stability conditions are presented by employing a parameter-dependent Lyapunov function method combined with average-dwell-time technique; and 3) a scheduling-and-feedback-control codesign procedure is proposed for the simultaneous stabilization of the collection of NCSs under consideration.

The rest of this paper is organized as follows. In Section II, a group of continuous-time LTI plants affected by network-induced delays are modeled as a collection of discrete-time polytopic uncertain systems with one step delay. In Section III, we give sufficient stability conditions for a single-control system and a schedulability condition for the considered NCSs, and then, simultaneous stability conditions are proposed by a parameter-dependent Lyapunov function method. In Section IV, a scheduling-and-feedback-control codesign procedure is developed for the simultaneous stabilization of the NCSs. Simulation studies are performed to demonstrate the effectiveness of the proposed codesign procedure in Section V. Finally, conclusions are included in Section VI.

II. NCS Model

In this paper, we consider the NCSs consisting of a collection of continuous-time LTI plants whose feedback-control loops are closed via a shared network link, as shown in Fig. 2. The ith plant, $i = 1, \ldots, N$, is given by

$$\dot{x}_i(t) = A_i^c x_i(t) + B_i^c u_i(t) \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ are the system states and $u_i(t) \in \mathbb{R}^{m_i}$ are the control inputs. The pair $(A_i^c \; B_i^c)$ is assumed to be stabilizable, but A_i^c might be unstable matrices.

In this paper, the following assumptions are made.

Assumption 1 [13], [15]: Because of the limitation of communication capacities, not all the control loops in the NCSs can be addressed at the same time. At any time instant, only $C_{\max}$ of the N plants ($C_{\max} < N$) can communicate with their remote controllers while others must wait, i.e., only $C_{\max}$ of the N plants can close their feedback loops at any time instant while the other control loops are assumed to be open loop.

Assumption 2 [13], [15]: When a plant fails to communicate with its corresponding controller, the open-loop system is unstable; otherwise, the plant communicates with its controller, and the resulting closed-loop system is stable.

Assumption 3 [23]: The following setup is considered in this paper: a clock-driven sensor, which periodically samples the plant outputs, an event-driven controller, which calculates the control signal as soon as the sensor data arrive, and an event-driven actuator, which updates the plant inputs as soon as the controller data arrive. The data are transmitted in a single packet at each time step.

Remark 1: To make the problem nontrivial, in Assumption 2, the open-loop systems are assumed to be unstable. In this paper, we only consider the worst case of unstable open-loop systems,

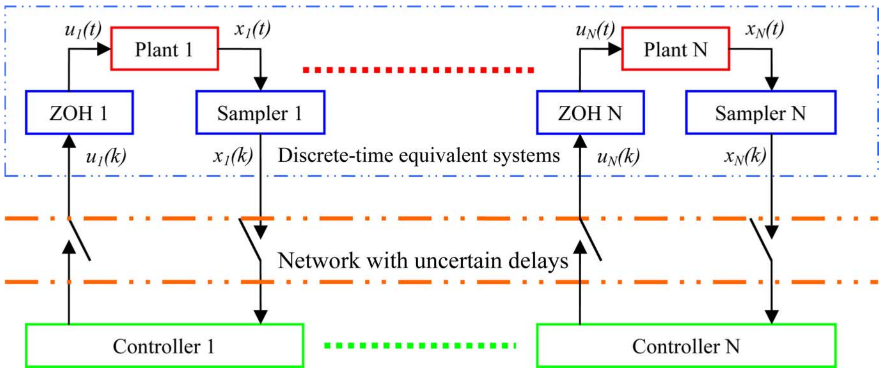

Fig. 2. Communication of controllers and plants through a shared network link.

and it should be mentioned that stable open-loop systems will be special cases in our discussion. This is, the results of this paper will still hold when the open-loop systems are stable, while the results may be conservative for the special cases.

In order to design a computer-based control, a sampled model of the continuous-time plant is used, which is based on [36]. Assume that the constant sampling period of the ith plant is h_i; the equivalent discrete representation of plant (1) is given by

$$x_i((k+1)h_i) = \Phi_i x_i(kh_i) + \Gamma_i u_i(kh_i) \tag{2}$$

where $\Phi_i = \mathrm{e}^{A_i^c h_i}$, $\Gamma_i = \int_0^{h_i} \mathrm{e}^{A_i^c s} ds B_i^c$, and $k \in \mathbb{Z}^+$ (nonnegative integers).

Throughout this paper, we use $x_i(k+1)$, $x_i(k)$, and $u_i(k)$ to denote $x_i((k+1)h_i)$, $x_i(kh_i)$, and $u_i(kh_i)$, respectively, for convenience.

A more realistic discrete representation should consider transmission delays in a data network. There are two sources of delays in the network [23]: the sensor-to-controller delay $\tau_{\mathrm{isc}}(k)$ and the controller-to-actuator delay $\tau_{\mathrm{ica}}(k)$. For static control law, the sensor-to-controller and controller-to-actuator delays can be lumped together as $\tau_i(k) = \tau_{\mathrm{isc}}(k) + \tau_{\mathrm{ica}}(k)$. Now, we assume that $\tau_i(k)$ is unknown time varying and satisfies

$$0 \le d_{i1} \le \tau_i(k) \le d_{i2} < h_i \tag{3}$$

where d_{i1} and d_{i2} are constant positive scalars representing the lower and upper bounds of delays, respectively.

Sampling system (1) with period h_i gives the following discrete representation [36]:

$$x_i(k+1) = \Phi_i x_i(k) + \Gamma_i^0(\tau_i(k)) u_i(k) + \Gamma_i^1(\tau_i(k)) u_i(k-1) \tag{4}$$

where $\Phi_i = \mathrm{e}^{A_i^c h_i}$ and

$$\Gamma_i^0(\tau_i(k)) = \int_0^{h_i - \tau_i(k)} \mathrm{e}^{A_i^c s} ds B_i^c \tag{5}$$

$$\Gamma_i^1(\tau_i(k)) = \int_{h_i - \tau_i(k)}^{h_i} \mathrm{e}^{A_i^c s} ds B_i^c = \Gamma_i - \Gamma_i^0(\tau_i(k)) \tag{6}$$

with $\Gamma_i = \int_0^{h_i} \mathrm{e}^{A_i^c s} ds B_i^c$.

Note that $\Gamma_i^0(\tau_i(k))$ and $\Gamma_i^1(\tau_i(k))$ are dependent on the unknown time-varying delay $\tau_i(k)$, and thus, system (4) is an uncertain linear system with time-varying uncertainties. Note that $\Gamma_i^0(\tau_i(k))$ cannot take any value; they belong to the convex hulls [20], [21], [23]

$$\Gamma_i^0(\tau_i(k)) \in \mathrm{co}\{G_{i1}, \ldots, G_{ir}\} \tag{7}$$

where $\mathrm{co}\{\cdot\}$ denotes the convex hull and constant matrices $G_{i1}, \ldots, G_{ir}$ represent the vertices of the hulls.

Remark 2: There are some approaches which can be used to obtain vertices of the convex hull (7) in the NCS literature, e.g., [20], [21]. Using the Taylor series expansion, in [20], a discretized system with a time-varying network-induced delay can be approximated by a polytopic uncertain system. A method for calculating the vertices of polytopic systems was developed in [21] by the real Jordan form for the representation of NCS with uncertain delay. In simulation studies, we will apply one of them to derive the convex hull vertices $G_{i1}, \ldots, G_{ir}$.

Consider system (4) with (5), (6), and (7) and static state-feedback laws $u_i(k) = K_i x(k)$; thus, the closed-loop system is given by

$$x_i(k+1) = \mathcal{A}_{ci} x_i(k) + \mathcal{B}_{ci} x_i(k-1) \tag{8}$$

where

$$\begin{aligned} \mathcal{A}_{ci} &= \sum_{j=1}^{r} \theta_{ij}(k) A_{cij} \qquad & \mathcal{B}_{ci} &= \sum_{j=1}^{r} \theta_{ij}(k) B_{cij} \\ A_{cij} &= \Phi_i + G_{ij} K_i \qquad & B_{cij} &= \Gamma_i K_i - G_{ij} K_i \\ \sum_{j=1}^{r} \theta_{ij}(k) &= 1 \qquad & \theta_{ij}(k) &\ge 0. \end{aligned}$$

It is clear that system (8) is a discrete-time polytopic uncertain system with one step delay. The discretizing continuous-time systems affected by uncertain network-induced delays are represented as system (8). It should be pointed out that this modeling method might be conservative, particularly in the case where the delay has very large values, but this does not occur frequently. From the control point of view, the digital control synthesis of continuous-time systems affected by time-varying delays is a challenge problem [20]. In this paper, we try to solve the control synthesis problem for the system with uncertain network-induced delays from the robust control point of view.

The objective of this paper is to design a scheduling policy and state-feedback-control laws $u_i(k) = K_i x(k)$ such that all systems in (8) are exponentially stabilized.

III. Stability for NCSs

A. Single-Control Loop Stability Analysis

In this section, the subscript i will be dropped for short. We use $[0\ k)$ to denote the time interval $[0\ kh_i)$, where h_i is the sampling period of the ith plant. Throughout this paper, the subscripts "c" and "o" stand for closed-loop and open-loop systems, respectively.

This section focuses on the stability of a single plant with closed/open feedback-control loop. As we know, a switched system may be unstable even if all its subsystems are stable, and it is still possible to be stabilized when all its subsystems are unstable by properly designed switching laws [17], [18], [37], [38]. Average-dwell-time technique [17] is an effective tool for designing such switching laws. As shown in [39], if the average dwell time is chosen sufficiently large and the total activation time of unstable subsystems is relatively small compared with that of stable subsystems, then exponential stability with a desired decay rate of the switched system is guaranteed. Motivated by the results, it is possible for a plant to be exponentially stabilized if the plant closes its feedback control for sufficiently large average dwell times and the total activation time of its open loop is relatively small compared with that of its close loop. Here, we are interested in solving the following problem: how often and how long one should close the control loop such that the control system is stable.

It is supposed that the single-control system is an open loop for some time because the shared network link is occupied by another network user. The single-control system can be described by a switched system, which is composed of the open-loop subsystem

$$x(k+1) = A_o x(k) \tag{9}$$

and the closed-loop subsystem

$$x(k+1) = \mathcal{A}_c x(k) + \mathcal{B}_c x(k-1) \tag{10}$$

where $A_o = \Phi = \mathrm{e}^{A^c h}$ and $\mathcal{A}_c$ and $\mathcal{B}_c$ are defined in (8).

For stability analysis, the following definitions play crucial roles in the sequel.

Definition 1: The system $x(k+1) = f(x(k))$ with $f(0) = 0$ is said to be exponentially stable with decay rate $0 < \rho < 1$ if $\|x(k)\| \leq c\rho^{k-k_0}\|x(0)\|\ \forall k \geq k_0$ holds for a constant $c > 0$.

Definition 2 [16]: For any $k > 0$, let $\alpha_{ci}(k)$ denote the total number of sampling periods of the ith plant being a closed loop (attended by the controller) during $[0\ k)$, and the ratio $\alpha_{ci}(k)/k$ is said to be the attention rate of the ith plant, and let $N_i(k)$ denote the total number of switching for the ith plant between open- and closed-loop statuses, which is said to be the attention frequency.

For the single-control system composed of subsystems (9) and (10), choose a piecewise quadratic Lyapunov-like function candidate

$$V(k) = \begin{cases} V_c(k), & \text{if closed loop} \\ V_o(k), & \text{if open loop.} \end{cases} \tag{11}$$

The following result gives the exponential stability of a single-control system.

Lemma 1: Consider the single-control system composed of the unstable open-loop subsystem (9) and the stable closed-loop subsystem (10). The single-control system is exponentially stable with decay rate $0 < \rho < 1$ if the following conditions hold.

1) The positive definite quadratic functions $V_c(k)$ and $V_o(k)$ in (11) satisfy

$$V_c(k+1) \leq \lambda_c V_c(k) \qquad V_o(k+1) \leq \lambda_o V_o(k) \tag{12}$$

where $0 < \lambda_c < 1$ and $\lambda_o > 1$.

2) There exists a constant scalar $\mu > 1$ such that

$$V_c(k) \leq \mu V_o(k) \qquad V_o(k) \leq \mu V_c(k) \tag{13}$$

for any $x(k)$.

3) The attention rate satisfies

$$\frac{\alpha_c(k)}{k} \geq \frac{\ln\lambda_o - \ln\lambda^*}{\ln\lambda_o - \ln\lambda_c}. \tag{14}$$

4) The attention frequency satisfies

$$N(k) \leq N_0 + \frac{k}{T_a}, \qquad N_0 = \frac{\ln c}{\ln\mu}$$
$$T_a > T_a^* = \frac{\ln\mu}{2\ln\rho - \ln\lambda^*} \tag{15}$$

where T_a and N_0 are said to be the average dwell time and the chatter bound, respectively, $\lambda_c < \lambda^* < \rho^2 < 1$, and $c > 0$.

Proof: See Appendix A. ■

Remark 3: It is worth making a few remarks about this lemma. First of all, condition 1) of Lemma 1 implies that the function $V_c(k)$ in (11) along the state trajectory of the closed-loop subsystem (10) has an exponential decay property: $V_c(k) \leq \lambda_c^{(k-k_0)} V_c(k_0)$, where $0 < \lambda_c < 1$ and k_0 is the initial time step. Moveover, $V_o(k+1) \leq \lambda_o V_o(k)$ means that $V_o(k) \leq \lambda_o^{(k-k_0)} V_c(k_o)$ $(\lambda_o > 1)$, which gives an exponential increase of $V_o(k)$ along the state trajectory of the open-loop subsystem (9). Condition 2) first appeared in [40] and has almost become a standard in applying the average-dwell-time technique to design switching laws for switched systems [17], [39], [41], [42]. This condition restricts the class of applicable Lyapunov-like functions by requiring the existence of a maximal global constant ratio among the functions. Quadratic functions are universally considered in linear systems, and in this case, the existence of a global constant μ is automatically guaranteed [42]. To ensure the exponential stability of the system, condition 3) implies that the attention rate of the ith plant is required to be sufficiently large, while the attention frequency is restricted with condition 4).

B. Scheduling Policy for NCSs

In accordance with Assumptions 1 and 2, if some plants and their controllers monopolize the network, then the other plants might not be stabilizable. In order to achieve simultaneous stabilization for all the control systems, it is necessary to carefully schedule the communication tasks for the collection of NCSs.

This section concentrates on finding a scheduling policy for establishing and terminating communication between each system and its controller in a way that stabilizes all systems.

Lemma 2: Under Assumptions 1 and 2, consider the collection of NCSs with communication constraints as shown in Fig. 2. Suppose that conditions 1) and 2) in Lemma 1 hold for any single-control system, and the following condition holds:

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{\max} \tag{16}$$

where $0 < \lambda_{ci} < 1$, $\lambda_{oi} > 1$, $C_{\max} < N$, and $C_{\max}$ denotes the maximum number of plants which can communicate with their remote controllers at any time instant. Then, there exists a scheduling policy which guarantees the exponential stabilization of each system (8).

Proof: The proof is inspired by the continuous-time one in [16]. From (16), it is clear that there exists a positive scalar $\bar{\varepsilon}$ such that the following inequality:

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} + \varepsilon C_{\max} \le C_{\max}$$

holds for $0 < \varepsilon \le \bar{\varepsilon} < 1$. Let $\bar{\varepsilon} = 1 - 1/C_{\max}\sum_{i=1}^{N}\ln\lambda_{oi}/(\ln\lambda_{oi} - \ln\lambda_{ci})$. Then, we have

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} + \sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}}\varepsilon < \sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} + \varepsilon C_{\max} \le C_{\max}.$$

Therefore, we obtain

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi} + \varepsilon\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} = \sum_{i=1}^{N} \frac{\ln\lambda_{oi} - \ln\lambda_{oi}^{-\varepsilon}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{\max} \tag{17}$$

which holds for all $0 < \varepsilon \le \bar{\varepsilon} < 1$. Let $\lambda_i^* = \lambda_{oi}^{-\varepsilon}$ and $\rho_i^2 = \lambda_{oi}^{-\varepsilon/2}$. Note that $0 < \lambda_{ci} < 1 < \lambda_{oi}$ and (17); then, it is easy to verify that $0 < \lambda_{ci} < \lambda_i^* < \rho_i^2 < 1$ for $i = 1, \ldots, N$. Let

$$\beta_i = \frac{\ln\lambda_{oi} - \ln\lambda_i^*}{\ln\lambda_{oi} - \ln\lambda_{ci}}, \qquad i = 1, \ldots, N. \tag{18}$$

Then, we have $0 < \beta_i < 1$ and $\sum_{i=1}^{N}\beta_i < C_{\max}$.

Next, we propose a periodic scheduling policy which guarantees the asymptotic stability of the NCSs. It should be mentioned that the proposed discrete-time scheduling policy is motivated by the continuous-time one in [16].

1) Scheduling policy:

1) Choose $\mathcal{T} = \max_{1\le i\le N}\{L_i h_i\}$, where L_i is a positive integer sufficiently large to satisfy the average-dwell-time condition in (15) for the ith plant and h_i is the sampling period of the ith plant. For example, we may set $L_i = \lceil T_{ai}^*\rceil$, where T_{ai}^* is the lower bound of the average dwell time T_{ai} and $\lceil\cdot\rceil$ denotes the upper integer bound.
2) Close $C_{\max}$ control loops for their plants at any time instant. Activate the control loops from 1 to N in order, and let the ith control loop work for a time interval of length $\lceil\beta_i\mathcal{T}/h_i\rceil h_i$ for $i = 1, \ldots, N$.

Now, we show that, under the aforementioned scheduling policy, the NCSs are exponentially stable with decay rate ρ_i.

2) Stability verification: For any $t = kh_i > 0$, it can be written as $t = n\mathcal{T} + \Delta$, $0 \le \Delta < \mathcal{T}$, where n is a nonnegative integer and Δ is a real number.

For $i = 1$, the following two cases need to be considered.

1) If $\Delta < \lceil\beta_i\mathcal{T}/h_i\rceil h_i$, then $\alpha_{ci}(k) = n\lceil\beta_i\mathcal{T}/h_i\rceil + \lceil\Delta/h_i\rceil$ and $N(k) = n$.
   Then, we obtain
   $$\frac{\alpha_{ci}(k)h_i}{kh_i} = \frac{n\left\lceil\frac{\beta_i\mathcal{T}}{h_i}\right\rceil h_i + \left\lceil\frac{\Delta}{h_i}\right\rceil h_i}{n\mathcal{T}+\Delta} \ge \frac{n\beta_i\mathcal{T}+\Delta}{n\mathcal{T}+\Delta} \ge \frac{n\beta_i\mathcal{T}+\beta_i\Delta}{n\mathcal{T}+\Delta} = \beta_i$$
   and $k/T_a = kh_i/(T_a h_i) \ge t/\mathcal{T} \ge n$. Thus, it follows that $N(k) \le N_0 + k/T_a$.
2) If $\Delta \ge \lceil\beta_i\mathcal{T}/h_i\rceil h_i$, then $\alpha_{ci}(k) = n\lceil\beta_i\mathcal{T}/h_i\rceil + \lceil\beta_i\mathcal{T}/h_i\rceil$ and $N(k) = n+1$. Then, we have
   $$\frac{\alpha_{ci}(k)h_i}{kh_i} = \frac{n\left\lceil\frac{\beta_i\mathcal{T}}{h_i}\right\rceil h_i + \left\lceil\frac{\beta_i\mathcal{T}}{h_i}\right\rceil h_i}{n\mathcal{T}+\Delta} \ge \frac{n\beta_i\mathcal{T}+\beta_i\mathcal{T}}{n\mathcal{T}+\Delta} > \frac{n\beta_i\mathcal{T}+\beta_i\mathcal{T}}{n\mathcal{T}+\mathcal{T}} = \beta_i$$
   and $k/T_a = kh_i/(T_a h_i) \ge t/\mathcal{T} \ge n$. If set $N_0 \ge 1$ which yields $c > \mu$, then we have $N(k) = n+1 \le N_0 + k/T_a$.

From the aformentioned discussions and the definition of β_i in (18), it follows that

$$\frac{\alpha_{ci}(k)}{k} \ge \frac{\ln\lambda_{oi} - \ln\lambda_i^*}{\ln\lambda_{oi} - \ln\lambda_{ci}}$$

which satisfies condition 3) in Lemma 1. According to the selections for L_i in scheduling policy 1), conditions 3) and 4) in Lemma 1 are both satisfied for $i = 1$, under the proposed scheduling policy.

For $i > 1$, by shifting the initial time t_0 to the beginning of the first closed-loop sampling period of the ith plant and adjusting the initial state $x(0)$ correspondingly, it reduces to the case $i = 1$, and conditions 3) and 4) in Lemma 1 are both satisfied for the shifted ith plant. It is straightforward to show the exponential stability between the time-shifted (by a finite constant) control system and the original system. Therefore, all the systems are exponentially stable with a specified decay rate under the scheduling policy, which completes the proof. ■

Remark 4: The form of schedulability condition (16) is similar to the continuous-time one in [14] and [16]. According to Lemma 2, it is shown that the schedulability condition (16) only depends on the convergence rate λ_{ci} of the ith closed-loop system and the divergence rate λ_{oi} of the ith open-loop plant. To obtain a large value of N, it is desirable that λ_{ci} and λ_{oi} are small. Therefore, we should focus on designing state-feedback-control laws such that λ_{ci} are minimized in the sequel.

Remark 5: The proof of Lemma 2 employs a constructive method, which provides a systematic way to design a scheduling policy to guarantee the simultaneous stability of all N systems. The scheduling policy is a static periodic scheduling policy, which is quite simple and can be easily implemented in engineering practice [16].

C. Simultaneous Stability

In this section, simultaneous stability conditions are presented for the collection of networked LTI systems.

For the polytopic system (8) with closed/open control loops, we choose the following piecewise Lyapunov-like function candidate as

$$V_i(k) = \begin{cases} V_{ci}(k), & \text{if closed loop} \\ V_{oi}(k), & \text{if open loop} \end{cases} \tag{19}$$

where

$$V_{ci}(k) = x_i^{\mathrm{T}}(k)\mathcal{P}_{ci}x_i(k) + x_i^{\mathrm{T}}(k-1)\mathcal{Q}_{ci}x_i(k-1)$$
$$V_{oi}(k) = x_i^{\mathrm{T}}(k)P_{oi}x_i(k) + x_i^{\mathrm{T}}(k-1)Q_{oi}x_i(k-1)$$

with $P_{oi} > 0$, $Q_{oi} > 0$, $\mathcal{P}_{ci} = \sum_{j=1}^{r}\theta_{ij}(k)P_{cij}$, $\mathcal{Q}_{ci} = \sum_{j=1}^{r}\theta_{ij}(k)Q_{cij}$, $P_{cij} > 0$, and $Q_{cij} > 0$.

Remark 6: In (19), the Lyapunov function matrices $\mathcal{P}_{ci}$ and $\mathcal{Q}_{ci}$ are similar to the form of (10), which are time variant and dependent on uncertain parameters $\theta_{ij}(k)$, $(j = 1, \ldots, r)$. When $P_{ci1} = \cdots = P_{cir} = P_{ci} > 0$ and $Q_{ci1} = \cdots = Q_{cir} = Q_{ci} > 0$, $V_{ci}(k)$ in (19) shrinks to the single Lyapunov function $V_{ci}(k) = x_i^{\mathrm{T}}(k)P_{ci}x_i(k) + x_i^{\mathrm{T}}(k-1)Q_{ci}x_i(k-1)$. The use of the single Lyapunov function usually renders conservative stability conditions as shown in [43]. To improve the single Lyapunov function-based results, we therefore employ the parameter-dependent Lyapunov function (19) for the analysis and synthesis of the NCSs.

Theorem 1: Under Assumptions 1 and 2, consider the collection of NCSs with communication constraints as shown in Fig. 2 and let $u_i(k) = K_i x_i(k)$ be state-feedback laws. Given a positive integer $C_{\max} > 0$ and constants $0 < \lambda_{ci} < 1$, $\lambda_{oi} > 1$, and $\mu_i > 1$, suppose that there exist positive-definite matrices $P_{oi} > 0$, $Q_{oi} > 0$, $P_{cij} > 0$, $P_{cil} > 0$, $Q_{cij} > 0$, and $Q_{cil} > 0$, and matrices T_{cil} $(i = 1, \ldots, N;\ j, l = 1, 2, \ldots, r)$ satisfying (20)–(23), as shown at the bottom of the page. Then, there exists a scheduling policy which guarantees the exponential stabilization of each system (8), and the state decay estimations are given by

$$\|x_i(k)\| \le \sqrt{\frac{b_i c_i}{a_i}}\rho_i^k \|x_i(0)\|_\delta$$

where decay rates $0 < \rho_i < 1$, constants $c_i > 0$, $a_i = \min\{\lambda_{\min}(P_{cij})\ (j = 1, 2, \ldots, r),\ \lambda_{\min}(P_{oi})\}$, $b_i = \max\{\lambda_{\max}(P_{cij}) + \lambda_{\max}(Q_{cij})\ (j = 1, 2, \ldots, r),\ \lambda_{\max}(P_{oi}) + \lambda_{\max}(Q_{oi})\}$, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a symmetric matrix, respectively, and $\|x_i(0)\|_\delta = \sup\{\|x_i(-1)\|,\ \|x_i(0)\|\}$.

Proof: See Appendix B. ■

Although exponential stabilization conditions for each discrete-time system (8) have been given in Theorem 1, we still cannot conclude that the each original continuous-time system (1) is asymptotically stable. The following result shows that the exponential stabilization of each system (8) implies simultaneous stabilization of the original collection of continuous-time linear plants.

Proposition 1: If conditions (20), (21), (22), and (23) in Theorem 1 hold, then each system (8) is exponentially stabilized, and its original continuous-time system (1) is asymptotically stable.

Proof: See Appendix C. ■

IV. Control-and-Scheduling Codesign

We shall devote this section to the design of state-feedback-control laws and scheduling policy for the NCSs.

If the gain matrices K_i and constants λ_{ci} are given, then inequality (20) is a linear matrix inequality (LMI) over the matrix variables P_{cij}, P_{cil}, Q_{cij}, and Q_{cil}, and matrices T_{cij}. However, since our purpose is to determine the gain matrices K_i, inequality (20) is actually a nonlinear matrix inequality. Note that inequality (20) implies $P_{cil} - T_{cil}^{\mathrm{T}} - T_{cil} < 0$, and T_{cil} is obviously nonsingular. By performing congruence transformation to (20) by $\mathrm{diag}\{T_{cil}^{-T}, I, I\}$, defining $\bar{T}_{cil} = T_{cil}^{-1}$ and $X_{cil} = P_{cil}^{-1}$, and using the Schur complement formula [44], we can thus conclude that inequality (20) is equivalent to (24) and (25), as shown at the bottom of the next page. To obtain the controller gain matrices K_i, we need to solve the feasibility solution to (24) and (25). Note that (25) is an equality constraint, and this problem is a nonconvex feasibility problem. Several approaches have been proposed to solve such nonconvex feasibility problem. One efficient way to solve the nonconvex problem is the cone complementarity linearization (CCL) algorithm proposed in [45], which has been used to solve the controller design problem, e.g., [24]. Now, using the CCL algorithm [45], the controller gain matrices K_i $(i = 1, \ldots, N)$ can be obtained by solving the following nonlinear minimization problem with LMI constrains:

$$\text{minimize Tr}\left(\sum_{l=1}^{r} P_{cil}X_{cil}\right)$$

$$\begin{bmatrix} P_{cil} - T_{cil}^{\mathrm{T}} - T_{cil} & T_{cil}(A_{oi} + G_{ij}K_i) & T_{cil}(\Gamma_i K_i - G_{ij}K_i) \\ (A_{oi} + G_{ij}K_i)^{\mathrm{T}}T_{cil}^{\mathrm{T}} & -\lambda_{ci}P_{cij} + Q_{cil} & 0 \\ (\Gamma_i K_i - G_{ij}K_i)^{\mathrm{T}}T_{cil}^{\mathrm{T}} & 0 & -\lambda_{ci}Q_{cij} \end{bmatrix} < 0 \tag{20}$$

$$A_{oi}^{\mathrm{T}}P_{oi}A_{oi} - \lambda_{oi}P_{oi} + Q_{oi} < 0 \tag{21}$$

$$P_{oi} \le \mu_i P_{cij} \qquad P_{cij} \le \mu_i P_{oi} \qquad Q_{oi} \le \mu_i Q_{cij} \qquad Q_{cij} \le \mu_i Q_{oi} \tag{22}$$

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{\max}. \tag{23}$$

subject to (24) and

$$\begin{bmatrix} P_{cil} & I \\ I & X_{cil} \end{bmatrix} \geq 0,\ l = 1,\ldots,r. \tag{26}$$

From Theorem 1 and the aforementioned minimization problem, we can outline a procedure for scheduling-and-feedback-control codesign for the NCSs.

Algorithm 1:

1) For any fixed i, minimize λ_{ci} and solve the ith state-feedback gain K_i. The minimization problem can be solved by the following iterative algorithm.
   - Step 1) Choose a sufficiently large initial λ_{ci} such that there exists a feasible solution to (24) and (26). Set $\lambda_{ci}^0 = \lambda_{ci}$.
   - Step 2) Find a feasible solution set $(X_{cil}^0, P_{cil}^0, P_{cij}^0, Q_{cil}^0, Q_{cij}^0, \bar{T}_{cil}^0, K_i^0,\ j,l = 1,\ldots,r)$ to (24) and (26). Set $k = 0$.
   - Step 3) Solve the following LMI problem:

     $$\text{minimize}\ \mathrm{Tr}\left(\sum_{l=1}^{r}\left(P_{cil}^k X_{cil} + X_{cil}^k P_{cil}\right)\right)$$

     subject to (24) and (26).

     Set $X_{cil}^{k+1} = X_{cil}$, $P_{cil}^{k+1} = P_{cil}$, $P_{cij}^{k+1} = P_{cij}$, $Q_{cil}^{k+1} = Q_{cil}$, $Q_{cij}^{k+1} = Q_{cij}$, $\bar{T}_{cil}^{k+1} = \bar{T}_{cil}$, and $K_i^{k+1} = K_i$.
   - Step 4) If inequality (20) is satisfied, then set $\lambda_{ci}^0 = \lambda_{ci}$ and return to Step 2 after decreasing λ_{ci} to some extent. If inequality (20) is not satisfied within a specified number of iterations, then exit. Otherwise, set $k = k + 1$ and go to Step 3.
2) Minimize λ_{oi} subject to (21).
3) Find constants $\mu_i > 1$ satisfying (22).
4) For given a positive integer $C_{\max} > 0$, calculate the maximal value of N subject to (23).
5) Design the following periodic static scheduling policy for NCSs.
   a) Choose $\mathcal{T} = \max_{\leq i \leq N}\{L_i h_i\}$, where $L_i = \lceil T_{ai}^* \rceil$ with T_{ai}^* being the lower bound of the average dwell time T_{ai}. It is clear that L_i satisfies the average-dwell-time condition for the ith plant.
   b) Close $C_{\max}$ control loops for their plants at any time instant. Activate the control loops from 1 to N in order and let the ith control loop work for a time interval of length $\lceil \beta_i \mathcal{T}/h_i \rceil h_i$ for $i = 1,\ldots,N$.

Conclusion: The simultaneous stabilization of the collection of NCSs with network-induced delay (3) and communication constraints, as shown in Fig. 2, can be obtained.

Remark 7: Algorithm 1 can also be applied in the design of static output-feedback laws $u_i(k) = K_i y_i(k)$, where $y_i(k) = C_i x_i(k)$ is the output of the ith plant. We can determine the static output-feedback gains K_i by replacing K_i in (24) with $K_i C_i$.

Remark 8: For Algorithm 1, we can give a conservative method for the estimates of constants μ_i

$$\mu_i = \max\left\{\frac{\lambda_{\max}(P_{cij})}{\lambda_{\min}(P_{oi})}, \frac{\lambda_{\max}(P_{oi})}{\lambda_{\min}(P_{cij})}, \frac{\lambda_{\max}(Q_{cij})}{\lambda_{\min}(Q_{oi})}, \frac{\lambda_{\max}(Q_{oi})}{\lambda_{\min}(Q_{cij})}\right\} \tag{27}$$

for $j = 1,2,\ldots,r$, which satisfies (22). However, this estimate of μ_i may result in a large average-dwell-time lower bound T_a^* from (15). In fact, there is another method, which will be used for simulation studies in Section V, for finding a proper μ_i. Solving Steps 1–4 in Algorithm 1 gives the parameterized matrices P_{cij} and Q_{cij}. Choose two small initial λ_{oi} and μ_i, then solve LMIs (21) and (22); if there is no feasible solution to (21) and (22), increase λ_{oi} or μ_i to some extent and solve LMIs (21) and (22) again, until (21) and (22) are feasible.

V. Simulation Studies

To show the proposed codesign procedure and validate its effectiveness, the control network setup in Fig. 1 is used for simulation studies, in which cart–pendulums are distributively controlled over CAN. Consider a cart–pendulum system, whose dynamics, taken from [46] and [47], can be described as

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$
$$\ddot{x}\cos\theta + \frac{4}{3}\ddot{\theta} - g\sin\theta = 0$$

where a force u is applied to the cart for keeping the pendulum balanced upright, x is the pendulum angular position, θ is the angle of the pendulum, M and m are the masses of the cart and the pendulum, respectively, l is the length of the pendulum, and g is the acceleration due to gravity. Selecting the state variables $\xi(t) = [\theta\ \dot{\theta}]^{\mathrm{T}}$ and linearizing the aforementioned model at the equilibrium point (i.e., $\xi(t)$=0) yield the following state-space model [47]:

$$\dot{\xi}(t) = \begin{bmatrix} 0 & 1 \\ \frac{3(M+m)g}{l(4M+m)} & 0 \end{bmatrix}\xi(t) + \begin{bmatrix} 0 \\ -\frac{3}{l(4M+m)} \end{bmatrix}u.$$

Let $m = 1.5306$ kg, $M = 5m = 7.6530$ kg, $l = 0.9333$ m, and $g = 9.8$ m/s^2. The simplified model of the cart–pendulum

$$\begin{bmatrix} -X_{cil} & \bar{T}_{cil}^{\mathrm{T}} & 0 & 0 \\ \bar{T}_{cil} & -\bar{T}_{cil} - \bar{T}_{cil}^{\mathrm{T}} & A_{oi} + G_{ij}K_i & \Gamma_i K_i - G_{ij}K_i \\ 0 & (A_{oi} + G_{ij}K_i)^{\mathrm{T}} & -\lambda_{ci}P_{cij} + Q_{cil} & 0 \\ 0 & (\Gamma_i K_i - G_{ij}K_i)^{\mathrm{T}} & 0 & -\lambda_{ci}Q_{cij} \end{bmatrix} < 0 \tag{24}$$

$$P_{cil}X_{cil} = I. \tag{25}$$

TABLE I
CALCULATE THE MINIMUM VALUES OF λ_c FOR DIFFERENT VALUES OF d_2 ($h = 0.04$ s AND $d_1 = 0$)

d_2	0.004	0.008	0.010	0.012	0.016
λ_c	0.5300	0.6885	0.7335	0.7655	0.8310

TABLE II
CALCULATE THE MINIMUM VALUES OF λ_c FOR DIFFERENT VALUES OF d_2 ($h = 0.06$ s AND $d_1 = 0$)

d_2	0.005	0.01	0.015	0.02	0.025
λ_c	0.4100	0.5690	0.6605	0.7005	0.9745

process is given by

$$\dot{\xi}(t) = A^c\xi(t) + B^c u(t) \tag{28}$$

where $A^c = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}$ and $B^c = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

The eigenvalues of matrix A^c are $\mathrm{eig}(A^c) = \{-3; 3\}$, which means that A^c is an unstable matrix. Now, consider the effect of transmission delays in the networks and suppose that $\tau_i(k)$ satisfies (3). We apply the procedure proposed in [21] to calculate the vertices of the convex hull (7), which are given by

$$\begin{aligned} G_1 &= -\frac{1}{18}\begin{bmatrix} \mathrm{e}^{-3(h-d_2)} + \mathrm{e}^{3(h-d_1)} - 2 \\ -3\mathrm{e}^{-3(h-d_2)} + 3\mathrm{e}^{3(h-d_1)} \end{bmatrix} \\ G_2 &= -\frac{1}{18}\begin{bmatrix} \mathrm{e}^{-3(h-d_2)} + \mathrm{e}^{3(h-d_2)} - 2 \\ -3\mathrm{e}^{-3(h-d_2)} + 3\mathrm{e}^{3(h-d_2)} \end{bmatrix} \\ G_3 &= -\frac{1}{18}\begin{bmatrix} \mathrm{e}^{-3(h-d_1)} + \mathrm{e}^{3(h-d_1)} - 2 \\ -3\mathrm{e}^{-3(h-d_1)} + 3\mathrm{e}^{3(h-d_1)} \end{bmatrix} \\ G_4 &= -\frac{1}{18}\begin{bmatrix} \mathrm{e}^{-3(h-d_1)} + \mathrm{e}^{3(h-d_2)} - 2 \\ -3\mathrm{e}^{-3(h-d_1)} + 3\mathrm{e}^{3(h-d_2)} \end{bmatrix}. \end{aligned}$$

Assuming that the lower delay bound of $\tau(k)$ is $d_1 = 0$, we are interested in the relationship between the minimum value of λ_c and the upper delay bound d_2. With different sampling periods $h = 0.04$ and $h = 0.06$, Tables I and II list the minimum values of λ_c for different d_2 by applying Steps 1–4 in Algorithm 1.

It is clear that the value of λ_c grows as d_2 increases from Tables I and II. Moreover, when $d_2 = 0$ (i.e., no networked-induced delay), λ_c approaches zero; hence, N tends to infinity from the schedulability condition (16). When $d_2 = 0.02$ (for $h = 0.04$) or $d_2 = 0.03$ (for $h = 0.06$), (20) is unfeasible.

Next, find constants λ_o and μ such that (21) and (22) hold. By Remark 8, there exists a feasible solution to (21) and (22) for a given $\mu = 4.2$ and $\lambda_c = 1.35$ ($h = 0.04$). Select constants $\mu = 6.02$ and $\lambda_c = 1.47$ ($h = 0.06$), (21) and (22) are feasible.

For simplicity, assume from now on that $C_{\max} = 1$, i.e., only one plant can close its feedback loop at any time instant.

Substituting $\lambda_o = 1.35$ and $\lambda_c = 0.7335$ into (23) gives

$$\tilde{\lambda} = \frac{\ln\lambda_o}{\ln\lambda_o - \ln\lambda_c} = 0.4919$$

which results in $2 * \tilde{\lambda} = 0.9839 < 1$ and $N = 2$ satisfying (23). Therefore, it can be concluded from Theorem 1 that such two identical systems with network-induced delays $0 \le \tau(k) \le 0.01$ and sampling period $h = 0.04$ can share a common network link, which only takes care of one control loop at a time. It follows by analogous analysis that three alike control loops can be scheduled under the assumption that $0 \le \tau(k) \le 0.004$ ($h = 0.04$). For sampling period $h = 0.06$ and $0 \le \tau(k) \le 0.0015$, two identical plants can be simultaneously stabilized via a shared common network link.

Then, we will focus on designing state-feedback laws $u_i(k) = K_i\xi_i(k)$ and a scheduling policy such that the following two control systems are exponentially stabilized.

A sampling system (28) with different periods ($h_1 = 0.04$ s and $h_2 = 0.06$ s) yields the following discrete representation:

$$\begin{aligned} \xi_1(k+1) &= \Phi_1\xi_1(k) + \Gamma_1^0(\tau_1(k))u_1(k) \\ &\quad + \Gamma_1^1(\tau_1(k))u_1(k-1) \qquad (29) \\ \xi_2(k+1) &= \Phi_2\xi_2(k) + \Gamma_2^0(\tau_2(k))u_2(k) \\ &\quad + \Gamma_2^1(\tau_2(k))u_2(k-1) \qquad (30) \end{aligned}$$

where Φ_i, Γ_i^0, and Γ_i^1 $(i = 1, 2)$ are defined in (4), respectively.

Assume the network-induced delay $0 \le \tau_i(k) \le 0.01$ $(i = 1, 2)$. Applying Algorithm 1 gives the following parameters:

$$\begin{aligned} &\lambda_{c1} = 0.7335 \qquad \lambda_{o1} = 1.35 \qquad \mu_1 = 4.2 \\ &K_1 = [151.187\ 32.0611] \end{aligned}$$

for system (29) and

$$\begin{aligned} &\lambda_{c2} = 0.5690 \qquad \lambda_{o2} = 1.47 \qquad \mu_2 = 6.05 \\ &K_2 = [116.7863\ 22.5267] \end{aligned}$$

for system (30). Hence, we have

$$\sum_{i=1}^{2} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} = 0.8979 < 1$$

which means that systems (29) and (30) can share a common network link. Let the desired exponential decay rates $\rho_1 = \rho_2 = 0.9988$. According to the proof of Lemma 2, select the following parameters: $\varepsilon = \bar{\varepsilon} = 0.1021$, $\lambda_1^* = 0.9698$, $\lambda_2^* = 0.9614$, $\beta_1 = 0.5422$, $\beta_2 = 0.4477$, $T_{a1}^* = 50.7743$, and $T_{a2}^* = 48.6986$. It follows from 5) of Algorithm 1 that $L_1 = 51$, $L_2 = 49$, $\mathcal{T} = 2.94$ s, $\lceil \beta_1\mathcal{T}/h_1 \rceil h_1 = 1.68$ s, and $\lceil \beta_2\mathcal{T}/h_2 \rceil h_2 = 1.32$ s. Therefore, the static period scheduling policy is determined. Close (30) first and activate controller $u_2(k)$ for the time interval of length 1.32 s, then close (29) and let its controller $u_1(k)$ work for the time interval 1.68 s, then, again, close (30) for the time interval of length 1.32 s. Hence, the time period of the scheduling policy is 3 s.

Let the initial states $\xi_1(0) = \xi_2(0) = [1, -2.5]$ and the network-induced delays $\tau_1(k) = \tau_2(k) = 0.01 - 0.01\sin(0.1k)$. Under the aforementioned period scheduling policy, the state trajectories of system (29) and the corresponding control input trajectory are shown in Figs. 3 and 4. Figs. 5 and 6 show the state and control input trajectories of system (30). Simulation

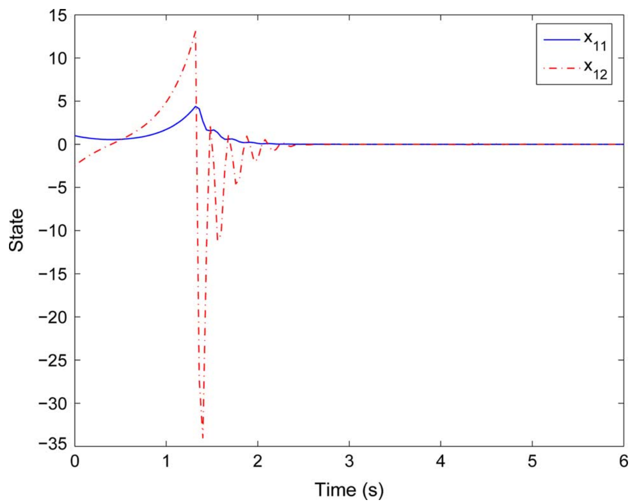

Fig. 3. State trajectories ($h_1 = 0.04$ s).

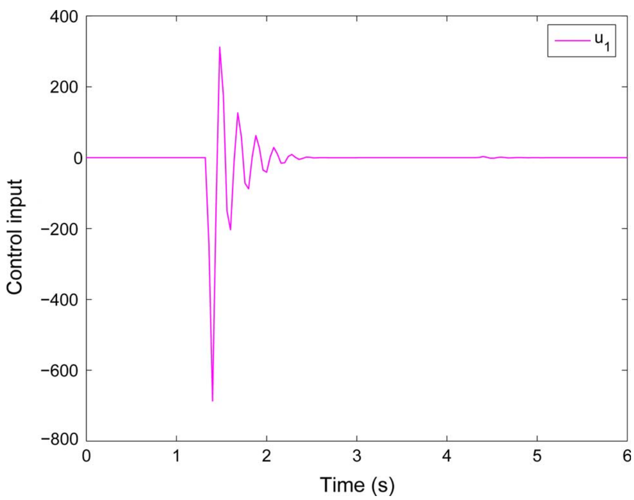

Fig. 4. Control input trajectory ($h_1 = 0.04$ s).

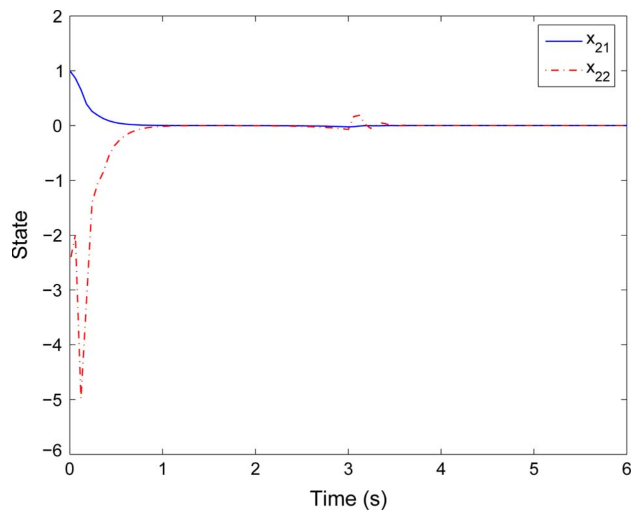

Fig. 5. State trajectories ($h_2 = 0.06$ s).

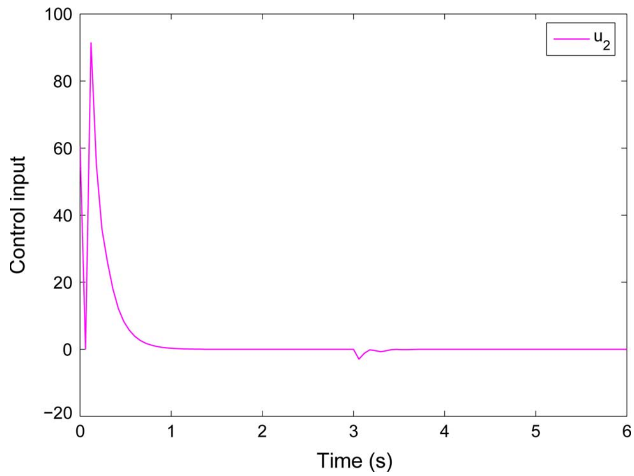

Fig. 6. Control input trajectory ($h_2 = 0.06$ s).

results demonstrate that the unstable open-loop systems are successfully stabilized by the proposed control laws and scheduling policy.

VI. Conclusion

In this paper, a scheduling-and-feedback-control codesign procedure has been proposed for the simultaneous stabilization of a collection of the plants whose feedback-control loops are closed via a shared communication network. The presence of feedback-based communication constraints and the effect of networked-induced delays have been taken into account in the communication network. Using average-dwell-time technique, stability conditions have been addressed for a single plant whose feedback-control loop is closed/open. Based on these stability conditions, a schedulability condition has been shown by a constructive proof which provides a systematic way to design a scheduling policy. The schedulability condition only depends on the convergence rate of the closed-loop system and the divergence rate of the open-loop plant. The effectiveness of the proposed codesign procedure has been illustrated through the application to the inverted pendulum control.

Controllability and reachability are two fundamental concepts in the analysis and design of control systems. The design of the communication sequences that preserve the controllability and reachability of a group of NCSs with communication constraints constitutes a challenging opportunity for future work. The ideas and algorithms presented in [48]–[50] for finding switching sequences to achieve the controllability and reachability of switched systems inspire us to design such communication sequences for the NCSs.

Appendix A
Proof of Lemma 1

Without loss of generality, we assume that the controller works during $[k_{2j}\ k_{2j+1})$ and that the plant is an open loop during $[k_{2j+1}\ k_{2j+2})$, $j = 0, 1, \ldots$, where $k_0 = 0$. Choose the piecewise quadratic Lyapunov-like function candidate as (11). For any $k > 0$, it holds from (12) that

$$V(k) \leq \begin{cases} \lambda_c^{k-k_{2j}} V_c(k_{2j}), & \text{if } k_{2j} \leq k < k_{2j+1} \\ \lambda_o^{k-k_{2j+1}} V_o(k_{2j+1}), & \text{if } k_{2j+1} \leq k < k_{2j+2} \end{cases} \quad (31)$$

where $0 < \lambda_c < 1$ and $\lambda_o > 1$.

Therefore, if $k \in [k_{2j+1}\ k_{2j+2})$, it follows from (13) and Definition 2 that

$$\begin{aligned} V(k) &\leq \lambda_o^{k-k_{2j+1}} V_o(k_{2j+1}) \\ &\leq \mu \lambda_o^{k-k_{2j+1}} V_c(k_{2j+1}^-) \\ &\leq \mu \lambda_o^{k-k_{2j+1}} \lambda_c^{k_{2j+1}-k_{2j}} V_c(k_{2j}) \\ &\leq \ldots \\ &\leq \mu^{N(k)} \lambda_c^{\alpha_c(k)} \lambda_o^{(k-\alpha_c(k))} V(0) \end{aligned} \tag{32}$$

where $N(k)$ and $\alpha_c(k)$ are defined in Definition 2 and k_{2j+1}^- denotes the time instant that is immediately before k_{2j+1}.

Similarly, we have $V(k) \leq \mu^{N(k)} \lambda_c^{\alpha_c(k)} \lambda_o^{(k-\alpha_c(k))} V(0)$ for $k \in [k_{2j}\ k_{2j+1})$.

From (14), we have

$$(\ln\lambda_o - \ln\lambda_c)\alpha_c(k) \geq (\ln\lambda_o - \ln\lambda^*)k$$

which is equivalent to

$$\lambda_c^{\alpha_c(k)} \lambda_o^{k-\alpha_c(k)} \leq (\lambda^*)^k. \tag{33}$$

From (15), we have

$$\begin{aligned} \mu^{N(k)} &\leq \mu^{N_0+(k/T_a)} = \mu^{N_0} \mu^{k/T_a} \\ &\leq \mu^{N_0} \mu^{k(2\ln\rho - \ln\lambda^*)/\ln\mu} = c\left(\frac{\rho^2}{\lambda^*}\right)^k. \end{aligned} \tag{34}$$

Combining (32), (33), and (34) yields

$$V(k) \leq c\rho^{2k} V(0). \tag{35}$$

If a quadratic form is considered in the piecewise Lyapunov-like function (11), then there exist $a_c > 0$, $a_o > 0$, $b_c > 0$, and $b_o > 0$ such that

$$\begin{aligned} a_c\|x(k)\|^2 &\leq V_c(k) & a_o\|x(k)\|^2 &\leq V_o(k) \\ V_c(0) &\leq b_c\|x(0)\|^2 & V_o(0) &\leq b_o\|x(0)\|^2 \end{aligned}$$

hold, which implies

$$a\|x(k)\|^2 \leq V(k) \qquad V(0) \leq b\|x(0)\|^2 \tag{36}$$

where $a = \min\{a_c, a_o\}$, $b = \max\{b_c, b_o\}$, and $\|\cdot\|$ denotes the Euclidean norm.

From (35) and (36), we have

$$\|x(k)\| \leq \sqrt{\frac{bc}{a}} \rho^k \|x(0)\|_\delta \tag{37}$$

which means that the single-control system is exponentially stable with decay rate $0 < \rho < 1$, where $\|x(0)\|_\delta = \sup\{\|x(-1)\|,\ \|x(0)\|\}$.

Appendix B
Proof of Theorem 1

The subscript i will be dropped for brevity in the proof. From Lemma 2, we know that there exists a scheduling policy that exponentially stabilizes all N systems if conditions 1) and 2) in Lemma 1 and (16) hold for every single-control system of the NCSs. It is obvious that (23) is the same as (16), and we shall prove that conditions 1) and 2) in Lemma 1 hold if (20), (21), and (22) hold for any fixed i.

First of all, it can be verified that $V_c(k+1) \leq \lambda_c V_c(k)$ holds according to (20) as follows.

Let $\mathcal{P}_c^+ = \sum_{j=1}^r \theta_j(k+1)P_{cj} = \sum_{l=1}^r \theta_l(k)P_{cl}$ and $\mathcal{Q}_c^+ = \sum_{j=1}^r \theta_j(k+1)Q_{cj} = \sum_{l=1}^r \theta_l(k)Q_{cl}$.

Define $\Delta V_c(k) = V_c(k+1) - V_c(k)$. From (19), the forward difference for $V_c(k)$ along the state trajectory of system (10) is given by

$$\begin{aligned} \Delta V_c(k) + (1-\lambda_c)V_c(k) &= V_c(k+1) - \lambda_c V_c(k) \\ &= \xi^{\mathrm{T}}(k)\Pi_c(\theta)\xi(k) \end{aligned} \tag{38}$$

where $\xi(k) = [x^{\mathrm{T}}(k)\ x^{\mathrm{T}}(k-1)]^{\mathrm{T}}$ and $\Pi_c(\theta)$ is as

$$\Pi_c(\theta) = \begin{bmatrix} \mathcal{A}_c^{\mathrm{T}}\mathcal{P}_c^+\mathcal{A}_c + \mathcal{Q}_c^+ - \lambda_c\mathcal{P}_c & \mathcal{A}_c^{\mathrm{T}}\mathcal{P}_c^+\mathcal{B}_c \\ \mathcal{B}_c^{\mathrm{T}}\mathcal{P}_c^+\mathcal{A}_c & -\lambda_c\mathcal{Q}_c + \mathcal{B}_c^{\mathrm{T}}\mathcal{P}_c^+\mathcal{B}_c \end{bmatrix}.$$

On the other hand, note that (20) implies $P_{cl} - T_{cl}^{\mathrm{T}} - T_{cl} < 0$ and that T_{cl} is obviously nonsingular. Since $P_{cl} > 0$, we have $(T_{cl} - P_{cl})P_{cl}^{-1}(T_{cl} - P_{cl})^{\mathrm{T}} > 0$, which is equivalent to

$$P_{cl} - T_{cl}^{\mathrm{T}} - T_{cl} > -T_{cl}P_{cl}^{-1}T_{cl}^{\mathrm{T}}. \tag{39}$$

From (20) and (39), we have

$$\begin{bmatrix} -T_{cl}P_{cl}^{-1}T_{cl}^{\mathrm{T}} & T_{cl}A_{cj} & T_{cl}B_{cj} \\ A_{cj}^{\mathrm{T}}T_{cl}^{\mathrm{T}} & -\lambda_c P_{cj} + Q_{cl} & 0 \\ B_{cj}^{\mathrm{T}}T_{cl}^{\mathrm{T}} & 0 & -\lambda_c Q_{cj} \end{bmatrix} < 0 \tag{40}$$

where A_{cj}, B_{cj}, and A_o are defined in (8) and (9), respectively.

Performing congruence transformation to (40) by $\mathrm{diag}\{T_{cl}^{-1}P_{cl}, I, I\}$ yields

$$\Xi_{cjl} = \begin{bmatrix} -P_{cl} & P_{cl}A_{cj} & P_{cl}B_{cj} \\ A_{cj}^{\mathrm{T}}P_{cl} & -\lambda_c P_{cj} + Q_{cl} & 0 \\ B_{cj}^{\mathrm{T}}P_{cl} & 0 & -\lambda_c Q_{cj} \end{bmatrix} < 0$$

which leads to

$$\begin{aligned} &\sum_{l=1}^r \sum_{j=1}^r \theta_l(k)\theta_j(k)\Xi_{cjl} \\ &\quad = \begin{bmatrix} -\mathcal{P}_c^+ & \mathcal{P}_c^+\mathcal{A}_c & \mathcal{P}_c^+\mathcal{B}_c \\ \mathcal{A}_c^{\mathrm{T}}\mathcal{P}_c^+ & -\lambda_c\mathcal{P}_c + \mathcal{Q}_c^+ & 0 \\ \mathcal{B}_c^{\mathrm{T}}\mathcal{P}_c^+ & 0 & -\lambda_c\mathcal{Q}_c \end{bmatrix} < 0. \end{aligned} \tag{41}$$

Let $\mathcal{J}_c = \begin{bmatrix} \mathcal{A}_c^{\mathrm{T}} & I & 0 \\ \mathcal{B}_c^{\mathrm{T}} & 0 & I \end{bmatrix}$. By pre- and postmultiplying (41) by $\mathcal{J}_c$ and its transpose, respectively, it follows that

$$\Pi_c(\theta) < 0 \tag{42}$$

where $\Pi_c(\theta)$ is defined in (38). From (38) and (42), we have

$$V_c(k+1) \leq \lambda_c V_c(k). \tag{43}$$

Therefore, we can conclude that $V_c(k+1) \leq \lambda_c V_c(k)$ if (20) holds.

Second, we shall verify that (21) results in $V_o(k+1) \leq \lambda_o V_o(k)$.

From (19) and (21), the forward difference for $V_o(k)$ along the state trajectory of system (9) is given by

$$\begin{aligned}\Delta V_o(k) + (1-\lambda_o)V_o(k) \\ =& V_o(k+1) - \lambda_o V_o(k) \\ =& x^{\mathrm{T}}(k)(A_o^{\mathrm{T}} P_o A_o - \lambda_o P_o + Q_o)x(k) \\ & -\lambda_o x^{\mathrm{T}}(k-1)Q_o x(k-1) \le 0\end{aligned}$$

which means that

$$V_o(k+1) \le \lambda_o V_o(k).$$

Therefore, we can conclude that condition 1) in Lemma 1 is satisfied if (20) and (21) hold.

Third, it follows from (19) and (22) that

$$P_o \le \mu \mathcal{P}_c \qquad \mathcal{P}_c \le \mu P_o \qquad Q_o \le \mu \mathcal{Q}_c \qquad \mathcal{Q}_c \le \mu Q_o$$

which implies that $V_c(k) \le \mu V_o(k)$ and $V_o(k) \le \mu V_c(k)$.

Lastly, for the considered Lyapunov-like function (19), it follows from (36) and (37) that

$$\|x(k)\| \le \sqrt{\frac{bc}{a}} \rho^k \|x(0)\|_\delta$$

where $0 < \rho < 1$, $c > 0$, $a = \min\{\lambda_{\min}(P_{cj})\ (j = 1, 2, \ldots, r),\ \lambda_{\min}(P_o)\}$, and $b = \max\{\lambda_{\max}(P_{cj})\ (j = 1, 2, \ldots, r),\ \lambda_{\max}(P_o)\}$. This completes the proof.

Appendix C
Proof of Proposition 1

To study the intersample behavior of the ith original continuous-time LTI system (1), the solution of the ith system (1) over the interval $t \in [kh_i,\ (k+1)h_i]$ is given by

$$\begin{aligned}x_i(t) =& \mathrm{e}^{A_i^c(t-kh_i)} x_i(k) \\ &+ \int_0^{t-kh_i-\tau_i(k)} \mathrm{e}^{A_i^c s} ds B_i^c K_i x_i(k) \\ &+ \int_{t-kh_i-\tau_i(k)}^{t-kh_i} \mathrm{e}^{A_i^c s} ds B_i^c K_i x_i(k-1).\end{aligned}$$

Let $\bar{\delta}_i = 1/2\lambda_{\max}(A_i^c + (A_i^c)^{\mathrm{T}})$. According to Lemma 5.1 [19], we have

$$\begin{aligned}\|x_i(t)\| \le \max\{\mathrm{e}^{\bar{\delta}_i h_i},\ 1\}\|x_i(k)\| + \bar{\delta}_i^{-1}(\mathrm{e}^{\bar{\delta}_i h_i - 1}) \\ \times \|B_i^c K_i\|(\|x_i(k)\| + \|x_i(k-1)\|)\end{aligned}$$

for $\bar{\delta}_i \ne 0$ and

$$\|x_i(t)\| \le (1 + h_i\|B_i^c K_i\|)\|x_i(k)\| + h_i\|B_i^c K_i\|\|x_i(k-1)\|$$

for $\bar{\delta}_i = 0$.

If conditions (20)–(23) in Theorem 1 hold, then each system (8) is exponentially stabilized, which implies that states $x_i(k)$ and $x_i(k-1)$ are bounded. Therefore, it is clear that $x_i(t)$ is also bounded. Moreover, the stability of each system (8) means that its states converge to zero as $k \to \infty$, which results in that $x_i(t)$ converges to zero as $t \to \infty$. Hence, the original continuous-time system (1) is asymptotically stable.

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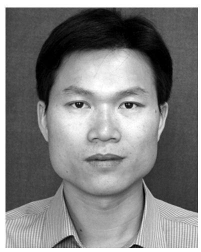

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