Problem 1: Consider the circuit shown below, which can be used as a model for a realistic inductor.

![Circuit Diagram]

Figure 1. Circuit model for problem 1.

a.) Find an expression for the impedance of this circuit.

b.) The frequency at which the impedance is purely resistive is called the resonant frequency. Under what conditions will there be a frequency (> 0) at which the impedance is purely resistive? Show that if this condition is satisfied, and if $CR^2/L << 1$ then the resonant frequency is given approximately by (hint: binomial expansion is helpful here):

$$\omega_p = \frac{1}{\sqrt{LC}} \left( 1 - \frac{CR^2}{2L} \right)$$

Note that this frequency is the parallel resonant frequency of the circuit, and that the effect of the resistance, R, is to make the parallel resonant frequency slightly smaller than that of the lossless (R=0) case.

c.) Starting from the exact impedance expression derived in part (a), if $L=100 \mu H$, $C=0.25$ pF, and $R=3$ Ω, graph the impedance. From your graph, what is an approximate circuit model for low frequencies (e.g. $\omega << \frac{1}{\sqrt{LC}}$)? Note: at very low frequencies, an “L” alone is enough; we are looking here for something a bit more sophisticated, but not as complex as the full model in Fig. 1 above. You may find that either a real/imaginary or magnitude/phase representation is easier to interpret.
Problem 2: A thin-film chip resistor listed in a component catalog as a “38.4 GHz” resistor has an equivalent circuit consisting of a 50 Ω resistance in parallel with an 83 fF capacitance (for fun, Google for it: USMRE1025T05FC). At what frequency is the impedance (magnitude) of this resistor different from the DC value by 10%? By 50%? If an application required a resistor with impedance magnitude within 5% of the DC value at 40 GHz, what “frequency rating” would be required?

Problem 3: In designing a multi-stage circuit, it is required to pass an RF signal with minimum loss but block the DC level. If the circuit board layout contributes 0.25 nH of effective series inductance, what size capacitor (assumed ideal, the capacitor’s size is < λ/100) would be needed to pass a signal at 5 GHz? What happens if this same capacitor is used in a system at 10 GHz?

Problem 4: For the experimental setup shown below, the oscilloscope trace shown below is obtained for a particular cable whose physical length is measured to be 1.25 m.

What is the velocity of propagation (in m/s) and its fractional velocity (speed relative to speed of light in a vacuum, c)? You should assume that the cables connecting the oscilloscope inputs and the power splitters are of the same length.

Input “A” on the oscilloscope is “vin” on the plot, input “B” on the oscilloscope is “vout” on the plot.