

Economics 70361: Problem Set 5
(due Monday, November 18)

Fall 2013

Professor Jensen

1. Two firms ($i = 1, 2$) engage in a race to win a patent. The value of the patent is $V > 0$. Let k_i be firm i 's flow effort on R&D. This effort purchases for firm i the hazard rate $h_i(k_i) = a_i k_i$, where $a_i > 0$. The flow cost of this R&D effort is $(1/2)c_i k_i^2$. Each firm's problem is to choose, at the beginning of the game, a value of k_i which will be its flow effort on R&D until one of the firm's succeeds in winning the patent. Time is continuous and the interest rate is $r \in (0, 1)$.

a. Determine the expected payoffs for each firm i as functions of k_1 and k_2 .

b. Let k_1^* and k_2^* be the equilibrium R&D efforts. Determine the signs of $\partial k_1^*/\partial a_1$, $\partial k_1^*/\partial a_2$, $\partial k_2^*/\partial c_1$, and $\partial k_2^*/\partial c_2$ (positive, negative, or ambiguous?). You may assume that the equilibrium is unique and locally stable. Provide economic interpretations for your answers.

2. Now suppose the firms agree to cooperate in R&D, and choose their flow costs to maximize the sum of their expected payoffs.

a. Let k_1^c and k_2^c be the optimal R&D efforts in this scenario. What conditions define k_1^c and k_2^c (i.e., what are the FONC of this problem)?

b. Can you rank k_i^* and k_i^c for each $i=1, 2$?

c. In which scenario is the expected discovery date lower?

d. Interpret the results on parts b and c.