University-Industry Spillovers, Government Funding, and Industrial Consulting

Abstract

We model of faculty consulting in the context of government and industry sponsored research. This frames an empirical analysis of the funding and consulting of individual faculty. In the theory, firms realize that they free ride on government sponsored research of their consultants and faculty realize their university projects indirectly benefit from their consulting. The model accounts for faculty quality, project characteristics, university policies, and the university's research support. Empirically, government funding is positively related to consulting, independent of faculty quality. Government and industry sponsored research funding act as strategic complements, and universities leverage their research infrastructure to attract funding.

1 Introduction

The existence of spillovers is one of the classic arguments for government research funding (Arrow 1962). This was clearly behind the National Science Council's call for increased federal funding for basic research along with increased actions by industry and the academy to encourage greater "intellectual exchange" between industry and academic institutions (National Science Board 2008). Unfortunately our understanding of such exchange is limited. While we know that academic research spills over to industry, we know little about the mechanisms involved (Jaffe 1989; Thursby and Thursby 2006). Much of our understanding comes from the analysis of spillovers associated with publications or patents (Adams 1990; Jaffe *et al.* 1993). Except for studies of licensing or start-ups from universities, there is little modeling of the actual mechanisms behind such spillovers (Zucker *et al.* 1998; Thursby and Thursby 2008).

In this paper, we examine industrial consulting by university faculty as a mechanism for spillovers in a context that allows us to link them to government, industry, and university funding decisions. The few existing studies of consulting suggest that it is an important factor in the intellectual exchange between the academy and industry. For example, Cohen *et al.* (1998) find that industrial managers often consider consulting to be more important than either patents or licensing for transferring knowledge from universities. In his survey, Mansfield (1995) found the reverse spillover (from firm projects to university research) provided benefits for faculty members' government supported research.

We develop a theoretical model of consulting which incorporates faculty decisions on the allocation of research time between their university lab and a firm's lab, as well as government and firm decisions on funding for the researcher's work within the university. We then exploit a unique database of funding, publications, and patents for 458 faculty inventors to estimate parameters of the model. The use of individual data, combined with the theory, provides new insights on the interface between university and industry research. In particular, we find evidence that government research funding increases faculty consulting on firm research projects, independent of faculty quality, and that government and industry funding for faculty research within the university are strategic complements (again independent of quality effects).

The model has two stages. In the first stage, a government agency and firm simultaneously choose funding levels for a researcher's university research project. This is followed by another simultaneous-move game in which the firm chooses a unit consulting fee, and the faculty researcher decides how much time to consult for the firm on its project. The model yields predictions for the time spent consulting and the associated fee and the level of government and industry support for university research.

We allow for differences in faculty quality and/or academic reputation, as well as differences in the scientific merit of projects within the university and firm. Research on both projects is uncertain. The firm can benefit from university research in several ways. It can license results from a successful university research project, but regardless of success or whether the firm funded university research, it can benefit from the researcher's expertise if it hires her to consult. Thus we allow for R&D spillovers in the sense that the researcher's work on government funded research can enhance her probability of success in the firm's consulting project. We also incorporate the notion that consulting on the firm's project can provide insights of use in her university research. Finally, the university supports the researcher's internal research through its infrastructure or some base level of funding.

The faculty researcher cares about reputation as well as income, so that the amount of time that she is willing to consult can be increasing or decreasing in the fee depending on her tradeoff between income and reputation, as well as her attitude toward risk. While in general the model's predictions depend on this relationship, several results hold regardless of the consulting supply function. In particular, increases in either the license revenue the university receives from her university project or her share of that revenue lead her to spend less time consulting.

The university-industry spillover allows the firm to free ride on university infrastructure. The effects of these spillovers, as well as the effects of changes in government and industry funding depend on the slope of the consulting supply function. However, in the absence of the university-industry spillover, increases in government funding and the university's internal research support reduce the time spent consulting regardless of the slope. It is this result that we exploit in our empirical analysis to test for such spillovers. Conversely, the reverse (industry-university) spillover allows the researcher to free ride on firm infrastructure in her university project, which leads to less consulting and a higher fee in equilibrium.

In the funding stage, obtaining unambiguous results requires additional assumptions in large part because of the ambiguous effects of funding on consulting in the presence of spillovers. Thus, in general, government and firm funding for research within the university can be strategic substitutes or complements. Nonetheless, if an increase in either type of funding increases the marginal effect of the other on the probability that the researcher's university project will be successful, then we can provide sufficient conditions for government and firm funding to act as strategic complements. In this case, an increase in university internal research support, license revenue, or the researchers share of license revenue all lead to increased government and firm funding for the researcher in equilibrium.

It is important to note that, while we assume the university project is more basic than the firm's, we follow Mansfield (1995) in focusing on consulting projects with some scientific merit. It is this feature of our analysis that allows us to overcome a major barrier to examining consulting empirically - that is, a lack of data on consulting time or fees. We employ a unique data set of nearly 1679 patents on which 458 faculty from eight major US universities are listed as inventors. Thirty percent of these patents are assigned to firms. In interviews with faculty and university licensing professionals, as well as industry R&D executives, the major reason given for faculty patents assigned to firms was consulting (Thursby *et al.* 2009). We exploit this information and use firm-assigned faculty patents as a measure of consulting activity. This measure captures only a subset of consulting outcomes by omitting projects that do not result in patents, but is important for our purposes because those consulting projects that result in patents clearly have scientific merit. Our data also include each individual's government and industry research funding by year, which allows us to provide estimates for both stages of the model.

In general, the empirical results support the theory. Results for the consulting stage support our assumptions that university research projects are more basic than firm projects and that there are spillovers from the researcher's university project to firms. We find that consulting is positively associated with government funding and our measures of university research support. In the context of the model, this result occurs only with spillovers from the faculty researcher's government sponsored research to the firm's research. In the funding stage, we find that government and industrial funding are strategic complements. In this case, the model implies that universities can use their research infrastructure to increase both types of funding, and indeed we find strong empirical support for this effect.

This is one of only a few studies to examine consulting either theoretically or empirically. The aforementioned surveys by Mansfield (1995) and Cohen *et al.* (1998) are notable exceptions. To our knowledge, the only theoretical studies of consulting are Beath *et al.* (2003) which examines the potential for budget-constrained universities to relax the constraint by encouraging faculty to consult and Dechenaux *et al.* (2009) which examines consulting as one of the mechanisms for inducing faculty inventors to collaborate in development needed for commercial success of inventions licensed from the university (Jensen and Thursby 2001). The latter differs markedly from this paper since the consulting considered is *ex post* development from a project started in the university rather than *ex ante* research by a faculty member on an industrial project.

The license share result in the theory is of particular note since it contributes to policy debates on the impact of licensing and other commercial opportunities on faculty research. The policy concern is that the opportunity to earn license revenue would divert faculty into applied work or research with little scientific merit. Empirical research examining this issue has been unable to find such an effect and in some cases, has found increased research in response to licensing (Azoulay *et al.* 2007, 2009; Thursby and Thursby 2009b). Our theoretical result provides a rationale for these findings since an increased share of license revenue will increase time spent on the university research project as well as funding for the research. Moreover, if the university research project has more scientific merit than the firm's, the increased share actually leads to an increase in fundamental research.[1]

We also contribute to the literature on the relationship between government and industry funding of research, which has primarily focused on the complementarity or substitutability of public and private funding of R&D conducted by firms (David and Hall 2000; David *et al.* 2000). We focus on funding for research *within* universities, and show that government and industry funding are strategic complements. In our model, this occurs because each type of funding improves the marginal impact of the other in the presence of spillovers, independent of the faculty member's reputation or quality. Thus our results differ markedly

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[1] Increased basic research in response to license incentives can be shown in other contexts (Jensen *et al.* 2003; Thursby *et al.* 2007; and Lach and Schankerman 2008).

from empirical studies of university level data which interpret a positive relationship between government and other funding as the result of the other sources taking federal funding as a signal of high quality research (Connolly 1997; Blume-Kohaut *et al.* 2009). Neither our theory nor empirics rely on signaling.

Finally, we contribute to the literature on spillovers from university research to industrial patenting. Our empirical approach of identifying faculty contributions to industrial patenting by firm-assigned patents on which faculty are inventors shows that spillovers are greater than those identified by the common practice of examining citations in firm-assigned patents to university-assigned patents (Jaffe 1989; Jaffe *et al.* 1993; Henderson *et al.* 1998).[2] In this regard, our work is closest to Thursby *et al.* (2009) which uses the same empirical approach to show that 26% of a sample of 5811 patents with faculty inventors from 87 US universities were assigned solely to firms. That work differs in that it is purely empirical and focuses on assignment as a function of patent and university characteristics rather than individual inventor characteristics or research funding.

Section 2 describes the research technology and player preferences and Section 3 presents the two stage game. Section 4 presents the econometric analysis. Section 5 concludes. Proofs of all propositions and the data description are given in Appendices A and B, respectively.

2 Environment

Our goal is to develop a theory to explain consulting as a mechanism for spillovers between university and industry research projects. For simplicity, we consider one faculty researcher, one firm interested in capitalizing on faculty expertise, and one government funding agency. We incorporate the fact that faculty differ in quality, which we represent by an observable variable q defined on the interval $[0, Q]$ such that higher values of q correspond to greater research capability.

2.1 Research Technology

Although there are many dimensions on which one can categorize research, for the purposes of this analysis it is most useful to think in terms of the pure scientific component of a given research problem. Thus, we assume research problems can be characterized by a variable x, defined on the interval $[0, X]$, such that higher values of x correspond to research that has greater scientific merit and is inherently more difficult to solve.

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[2] For a similar point in a European context see Crespi et al.(2006), Geuna and Nesta (2006), and Saragossi and van Pottelsberghe de la Potterie (2003).

Successfully solving a given research problem can generate multiple outputs of value to the researcher, university, government funding agency, and/or industrial sponsor. These can be generally thought of as those results of research that contribute to the scientific reputations and commercial payoffs associated with solving the problem, such as publications, citations, patents, and profits. The likelihood that a research project succeeds depends on a number of factors, including the nature of the problem to be solved (how fundamental or basic it is), the quality of the researcher, and the level of funding available.

For simplicity, we think of the researcher as working on a single research problem within the university, which has scientific merit x_I, with the possibility of also working outside the university as a consultant on a firm's research problem, which has scientific merit x_O, where $x_I > x_O$. While this assumption is not necessary for our results, it is consistent with the bulk of the literature on university industry collaboration (Rosenberg and Nelson 1994 and 1996; Mowery and Teece 1996; Lacetera 2009). Assume that T is the total time available in the period, and that M is the (maximum) amount of time that she can spend consulting, $M < T$.[3] If t is the time she contracts to consult with the firm, $t \in (0, M]$, then $T - t$ is the time she spends on her own research project in the university. If she does not consult, $t = 0$, then she works all of the time T on her own university research.[4]

We model research as an uncertain production process in which the "production function" is a probability of success function. We assume that the probability of success in solving any specific research problem of scientific merit x undertaken by a researcher of quality q is $p(\tau, e; q, x)$, where τ represents the time the researcher devotes to the project, and e represents her effective funding on that project. From the production perspective, it is natural to assume that p is increasing and strictly concave in (τ, e, q), so these "inputs" have positive but diminishing marginal productivities. It is also natural to assume that these inputs are complements, so the second order cross-partial derivatives of p with respect to them are all positive. For example, the marginal effect of an additional hour of research on the probability a project will succeed should be greater for researchers with higher quality or greater levels of funding. Our assumption that it is more difficult to solve problems with greater scientific merit implies p is decreasing in x, and it also natural to assume that this difficulty increases at a increasing rate, $\partial^2 p/\partial x^2 < 0$. We also assume that a more difficult

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[3] Most funding agencies and universities will not allow researchers to sell more than 100% of their time, so a decision to consult for the firm in its research lab on its project clearly means that the researcher will not be spending all of her time on her university project. Indeed, if she chose to so this after accepting, for example, federal funding for the entire year, then the granting agency would undoubtedly adjust their level of funding for her to adjust for this.

[4] This implicitly assumes that our heroine is an obsessive-compulsive workaholic who prefers her own research to all forms of leisure activity. This interpretation is perhaps an over-simplification, but it highlights the stylized fact that most researchers view their own research as a consumption good.

project reduces the marginal effect on the probability of success of time, effective funding, and quality, so that the cross-partial derivatives with respect to x and each of the inputs are negative.

It is important to discuss in more detail what we mean by “effective” research funding. A common approach to incorporate spillovers in models of R&D is to define effort as the sum of all expenditures on R&D, including spillovers (see DeBondt 1997). We take a similar approach and allow for spillovers between research projects by including effective funding in the probability of success functions.

The researcher has several sources of potential support for her university research. One is university infrastructure and knowledge base, K_I. She can also obtain sponsored research funds from a government agency, G, and/or industrial firm, F. Finally, if she consults for a firm, then there may be spillovers to her university research.

The firm’s research project is conducted within its own lab, where K_O represents its infrastructure and knowledge base. The unit cost of consulting paid by the firm is c, so ct is the firm’s expenditure on consulting (and the researcher’s consulting income)

Thus, we define effective funding on the researcher’s project in the university as

$$\begin{aligned} e_I &= K_I + G + F \; + \lambda K_0 \qquad if\ t > 0 \\ &= K_I + G + F \qquad\qquad\quad if\ t = 0 \end{aligned} \tag{1a}$$

where $\lambda \in [0,1)$ is the spillover parameter representing the extent to which her consulting experience can contribute to her university research. Spillovers from consulting to university research are a function of the firm’s knowledge base.

Analogously, we define effective funding for the firm’s project as

$$\begin{aligned} e_O &= K_O + F + \beta(K_I + G) + ct \qquad if\ t > 0 \\ &= K_O + F \qquad\qquad\qquad\qquad\quad if\ t = 0, \end{aligned} \tag{1b}$$

where $\beta \in [0,1)$ is the spillover parameter representing the extent to which her university research experience can contribute to solving the firm’s problem. This approach assumes that when the firm hires a university researcher as a consultant, what she contributes to the project is a function of the knowledge from her government and university supported research. It also embodies the idea that the firm funds university research both to obtain results with commercial potential and to enhance the firm’s own internal research problems (Thompson 2003).

2.2 Preferences and Payoffs

In general, we think of faculty as deriving utility from the prestige associated with their research as well as the income it generates (Stephan 1996). Therefore, we assume that faculty utility at any date is $U(R, W)$, where R is her current stock of academic (scientific) reputation and W is her current wealth stock.[5] Marginal utility in reputation is positive and diminishing, while marginal utility of wealth is positive and nondecreasing (we allow for the case of risk neutrality to clarify which results do not depend on risk-aversion). Let R_s denote her reputation if she successfully solves her university research problem in this period. We assume R_s is an increasing function of x_I, because successful solution of a research problem of greater scientific merit results in greater enhancement of her reputation.[6] Let R_f denote her reputation if she fails to solve the problem in this period. This is also her reputational stock at the beginning of the period, when the funding agency and firm make their funding decisions. Thus, conditional on success $R_s - R_f > 0$ is the flow of reputation in this stage (conditional on success). Define A as her wealth stock at the beginning of the period plus her university salary minus savings, that is her current net assets (i.e., net of savings and non-innovation income). Also assume that γ is her share of license revenue paid to the university for a success, $L \geq 0$. Then current wealth is $W_s = A + \gamma L + ct$ for success and $W_f = A + ct$ for failure, forms which emphasize the flow income from university invention and consulting. The researcher's expected utility is then

$$\begin{aligned} EU(G, F, t, c) &= p(T - t, e_I; q, x_I)U(R_s, W_s) \\ &\quad +[1 - p(T - t, e_I; q, x_I)]U(R_f, W_f). \end{aligned} \tag{2}$$

This approach allows us to focus on any given stage in the life cycle of this researcher. From her perspective, the results that follow depend on the stage of the life cycle only to the extent that they depend on the relative magnitudes of R and W.[7]

The government funding agency is primarily interested in advancing basic scientific research, so its utility, U_g, depends upon the scientific reputational stock associated with the research it has funded. Because there are alternative uses for its research budget, namely

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[5] An alternative approach would be to assume utility depends on the time spent in each type of research and the prestige of each type of research stock, as in Jensen and Thursby (2004). In the approach we take here, the time spent in each type of research effects her expected utility, but only through the probability of success.

[6] For notational convenience, we do not write this functional dependence explicitly except when necessary.

[7] This approach abstracts from the savings and salary determination decisions, but the additional complexity from endogenizing them would not add value to the analysis. As we show below, the stage of the life-cycle matters only to the extent that varying the relative stocks of R and W over time might change the sign of $\partial^2 U/\partial R \partial W$, and so possibly the direction of shift of her best reply function in response to changes in some of the parameters of the model in the consulting subgame.

other researchers' projects, its net expected utility, EU_g, from funding her project is the expected utility of its reputation less the utility loss V from not funding alternative projects. Its net expected utility from devoting G to this project is then

$$\begin{aligned} EU_g(G,F,t,c) &= p(T-t,e_I;q,x_I)U_g(R_{gs}) \\ &\quad +[1-p(T-t,e_I;q,x_I)]U_g(R_{gf})-V(G) \end{aligned} \tag{3}$$

where R_{gs} is its reputational stock if she succeeds in her university project, and R_{gf} is its reputational stock at the beginning of the period. We also assume R_{gs} is an increasing function of x_I. The agency does not get reputational credit for her success if it does not fund her: $R_{gs} > R_{gf}$ if and only if $G > 0$. Note that an increase in the researcher's consulting time unambiguously decreases the funding agency's expected utility by reducing the probability of success in her university project.

Finally, the firm's expected profit arises from both its own research problem and the university research that it funds. Let π_I denote firm profit from funding the researcher's university project if it succeeds and π denote the profit from its own research project if it succeeds. Then expected profit is

$$E\Pi(G,F,t,c) = p(T-t,e_I;q,x_I)(\pi_I - L) - F + p(t,e_O;q,x_O)\pi - ct. \tag{4}$$

This form implicitly assumes the firm will only fund her university project if it obtains an option to license a success from the project, and that it is not interested in licensing successful projects developed without its funding. This form also shows the trade-off in the probability of success of each project as her time spent consulting changes.

To save notation, we let p_I denote $p(T-t,e_I;q,x_I)$ and p_O denote $p(t,e_O;q,x_O)$ whenever their meanings are clear.

3 The Funding Game

We adopt a game structure that conforms to the stylized fact that faculty typically prefer their own research to other projects (Aghion *et al.* 2008), and therefore focus on obtaining funds for their research before making any agreements to consult on the firm's research project. Thus, the game has two stages. In the first stage, the researcher seeks support for her university research project from both the government funding agency and the firm. The agency and the firm then simultaneously choose funding levels for her project. After these decisions are made and revealed, but before the success or failure of the university research project is observed, another simultaneous-move game follows in which the firm chooses a unit consulting fee, and the researcher decides how much time to consult for the firm.[8]

Two comments about this approach are in order. First, it assumes that the funding agency and firm can pre-commit to providing funds for the researcher's university project.[9] It also assumes that researchers cannot be treated as agents who must accept take-it-or-leave-it offers. That is, we consider the behavior of scientists whose expertise and reputation gives them more "market power" than workers in a principal-agent model with a perfectly elastic supply of labor.

3.1 Stage Two Equilibrium

In the second stage the researcher chooses her consulting time t and the firm chooses its unit consulting fee c, given the values of funding for university research chosen in stage one, F and G. In Appendix A, we state and prove the formal result for existence of an equilibrium in this game, $(t^*(G,F), c^*(G,F))$.[10]

Choosing t to maximize $EU(G,F,t,c)$ yields a best reply function $\hat{t}(c)$ for the researcher, which defines the consulting time that maximizes her expected utility for any unit consulting fee chosen by the firm. This is essentially her consulting supply function. Similarly, choosing c to maximize $E\Pi(G,F,t,c)$ yields a best reply function $\hat{c}(t)$ for the firm[11], which defines the unit consulting fee that maximizes its expected profit for any time in consulting chosen by the researcher. This is essentially its (inverse) consulting demand function. The equilibria of this game are more easily understood using diagrams of these best reply functions (see Figures 1 and 2).

Because we are interested in deriving testable implications, we focus on the Nash equilibrium when it is interior,[12] in which case it must satisfy

$$\frac{\partial EU(G,F,t^*,c^*)}{\partial t} = 0, \tag{5a}$$

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[8] This approach also conforms to the "standard" academic year of nine months in which faculty are paid by the university, followed by three summer months in which faculty are free to pursue external funding options.

[9] This approach is similar to that in Lacetera (2009), who assumes that firms commit to university research as a way of funding basic research.

[10] These equilibrium values are also functions of all the parameters of the model $(\alpha,\beta,q,x_I,K_I,x_O,K_O,S,L,\gamma)$. Although a minor abuse of notation, we omit these as arguments of the functions for clarity of exposition.

[11] We omit the parameters of the model as explicit arguments of $\hat{t}(c)$ and $\hat{c}(t)$ for clarity of exposition.

[12] These results provide some information about corner solutions as well. For example, a change that increases consulting time in an interior equilibrium is more likely to induce a researcher off the no-consulting corner and begin some consulting.

and

$$\frac{\partial E\Pi(G,F,t^*,c^*)}{\partial c} = 0 \tag{5b}$$

where

$$\begin{aligned}\frac{\partial EU(G,F,t,c)}{\partial t} &= -\frac{\partial p_I}{\partial \tau}[U(R_s,W_s) - U(R_f,W_f)] \\ &\quad + c[p_I \frac{\partial U(R_s,W_s)}{\partial Y} + (1-p_I)\frac{\partial U(R_f,W_f)}{\partial Y}]\end{aligned} \tag{6a}$$

and

$$\frac{\partial E\Pi(G,F,t,c)}{\partial c} = \frac{\partial p_O}{\partial e_O} t\pi - t. \tag{6b}$$

In this case, the best reply functions are implicitly defined by setting (6*a*) and (6*b*) each equal to zero.

Examples of this equilibrium are depicted in Figures 1 and 2. To interpret the equilibrium conditions in (5), consider the expressions for marginal utility and marginal profit in (6). From (6*b*), an increase in the consulting fee increases effective funding e_O, and therefore increases the firm's expected profit, so it increases the fee as long it is less than the marginal increase in expected profit. Because the firm's best reply is its inverse demand function, we assume that

$$\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2} c < 0 \tag{7}$$

to insure that this demand curve, and the firm's best reply function, are negatively sloped. Further note that, because effective funding e_O also depends on funding for the researcher's university project, (6*b*) shows how spillovers from basic university research can influence the firm's unit consulting fee, and so whether our heroine actually consults.

Devoting more time to consulting has two conflicting effects on the researcher's expected utility. First, for any fee, more time in consulting increases her income, whether or not either research project succeeds, as shown by the second term in (6*a*). However, the first term in (6*a*) shows that diverting more time to consulting also decreases her expected utility by decreasing the probability of success in university research, and thus the probability of the resulting reputational enhancement and license revenue. If the expected loss of utility from diverting any time to consulting is too high, then she will not do so. Otherwise, she increases time in consulting until the marginal gain in expected utility from consulting income is offset by this marginal expected loss in her university research.

Appendix A shows that she does not consult for free. At $c = 0$, the expected marginal utility from consulting is negative, because diverting time from her university project decreases its probability of success and expected utility without providing any additional income in return, so $\hat{t}(0) = 0$. However, her expected marginal utility is increasing in the fee at $t = 0$, so $\hat{t}'(0) > 0$ and she may consult for a large enough fee. Because $\hat{t}'(c) = -\frac{\partial^2 EU}{\partial t \partial c} / \frac{\partial^2 EU}{\partial t^2}$, where

$$\begin{aligned}\frac{\partial^2 EU}{\partial t \partial c} &= [p_I - t\frac{\partial p_I}{\partial \tau}]\frac{\partial U(R_s, W_s)}{\partial Y} + \{1 - [p_I - t\frac{\partial p_I}{\partial \tau}]\}\frac{\partial U(R_f, W_f)}{\partial Y} \\ &\quad + ct[p_I \frac{\partial^2 U(R_s, W_s)}{\partial Y^2} + (1 - p_I)\frac{\partial^2 U(R_f, W_f)}{\partial Y^2}], \qquad (8)\end{aligned}$$

the slope of her best reply depends, in general, on both the elasticity of the probability of success in university research with respect to time spent consulting and her attitude toward risk.

Theorem 1 *When the researcher's best reply is interior, $\hat{t}(c) \in (0, M)$, her best reply function for consulting time is increasing if she is risk-neutral or not too risk-averse, and if the probability of success in her university research is not elastic with respect to the time diverted to consulting, $\frac{t}{p_I}\frac{\partial p_I}{\partial \tau} \leq 1$. However, it can be negatively sloped if this probability is elastic and/or she is sufficiently risk-averse.*

As we show in the appendix, consulting occurs in equilibrium only if her best reply is positively sloped initially (i.e., $\hat{t}'(c) > 0$ for the lowest c such that $\hat{t}(c) > 0$). It may remain positively sloped for all relevant fees. However, as the fee and her certain income increase, her best reply may eventually bend back, becoming negatively sloped thereafter. This can happen if the probability of success in her university research is elastic and/or she is sufficiently risk-averse. Therefore, at equilibrium, her best reply can be either positively or negatively sloped, as depicted in Figures 1 and 2.[13]

Comparative statics results depend not only on these slopes, but also on the direction in which the best-reply functions shift. For some parametric changes, the direction of shift in the researcher's best reply depends on how her marginal utility of income varies as a function of her academic reputation. To abstract from this question, we assume hereafter that her utility is additively separable, $U(R, W) = f(R) + g(W)$, where $f'(R) > 0 > f''(R)$ for all $R \geq 0$ and $g'(W) > 0 > g''(W)$ for all $W \geq 0$.[14] This allows us to state the comparative statics of the consulting subgame equilibria as follows.

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[13] There is, of course, the possibility that her best reply not only intersects the firm's when it is increasing, but also turns down so sharply that it intersects the firm's again from above. In this case, however, the latter equilibrium is not locally stable, so we do not consider it.

[14] As can be seen in Appendix A, this assumption can be replaced with the weaker assumption that $\frac{\partial U(R_s, W_s)}{\partial Y} \leq \frac{\partial U(R_f, W_f)}{\partial Y}$, which simply says that her marginal utility of income does not increase when her university research project succeeds.

Theorem 2 *In the equilibrium of the second stage consulting subgame:*
(i) An increase in license revenue L *or her share* γ *of it decreases consulting time and increases the consulting fee.*
(ii) An increase in the difficulty x_O *of the firm's project decreases the fee and decreases (increases) consulting if her best reply is positively (negatively) sloped.*
(iii) An increase in q *increases the fee and, if her best reply is negatively sloped, decreases consulting.*
(iv) An increase in government funding G*, industrial funding* F*, or the research support* K_I *provided by the university:*
(a) Decreases consulting but has an ambiguous effect on the fee if her best reply is positively sloped.
(b) Must either decrease consulting, decrease the fee, or both, if her best reply is negatively sloped.
(v) An increase in the research support K_O *provided by the firm has an ambiguous effect on fee, but decreases consulting if her best reply is positively sloped.*
(vi) An increase in her net assets A *increases consulting time and decreases the fee if she is risk neutral.*

These results are easily seen from the figures. An increase in either license revenue L or the researcher's share of it γ has no effect on the firm's best reply, but shifts hers to the left because she chooses to consult less for any given fee. Thus, in equilibrium, consulting time t^* decreases and the fee c^* increases. That is, our model predicts that the potential for income from their own university research would lead faculty to substitute time in university research for consulting. This result has important implications for the Bayh-Dole Act, which gave rights from governmentally funded patents to universities and their researcher-inventors, and so was equivalent to an increase in L and γ. Our analysis shows that, whatever the slope of the researcher's consulting supply, and whatever the stage of the life cycle, passage of this act would have reduced consulting. Although scholars and policy makers have expressed concern that this act may have led to less fundamental research, our analysis implies there is no reason to expect such an effect. This may explain the failure of empirical studies to find this effect (Azoulay *et al.* 2007, 2009; Thursby and Thursby 2009b). We emphasize that this result does not depend on the assumption that her utility is separable.

Next, an increase in x_O has no effect on the researcher's best reply, but the firm is willing to pay less per unit of time for her as a consultant, so its best reply shifts down. Consulting time decreases when her best reply is positively sloped, and increases when it is negatively sloped. In either case, the fee decreases. This result also does not depend on the assumption that her utility is separable.

An increase in her stock of assets A leads her to consult more for any fee if she is risk-neutral (or not too risk-averse), so her best reply shifts right. This has no effect on the firm's willingness to pay for her as a consultant, so consulting time increases and the fee decreases.

The results for G, F, K_I, and K_O assume that there are spillovers, $\lambda > 0$ and $\beta > 0$. We therefore defer this discussion until after the following result on spillovers.

Theorem 3 *In the equilibrium of the second stage consulting subgame:*
(i) An increase in the extent β to which her university research spills over into consulting decreases the fee, and decreases (increases) consulting, if her best reply is positively (negatively) sloped.
(ii) An increase in the extent λ to which consulting spills over into her university research decreases consulting time and increases the fee.
(iii) In the special case of $\beta = 0$, an increase in government funding G or the research support K_I provided by the university decreases consulting and increases the fee.
(iv) In the special case of $\lambda = 0$, an increase in the research support K_O provided by the firm in its lab decreases the fee and decreases (increases) consulting if her best reply is positively (negatively) sloped.

An increase in the extent of spillovers to consulting β has no effect on her best reply. The firm is willing to pay her less per unit of time because it benefits more by free-riding on university infrastructure, so its best reply shifts down. The effect depends on the slope of her best reply: consulting decreases, but the fee increases (decreases) when her best reply is negatively (positively) sloped. Because her best reply does not shift, this result does not depend on assuming that her utility is separable. An increase in the extent of spillovers from consulting λ has no effect on the firm's best reply. However, because her university research benefits from free-riding on the firm's infrastructure increase, she is willing to consult less for any given fee, so her best reply shifts left. Therefore, in equilibrium, consulting decreases and the fee increases.

It is important to understand how changes in the levels of government and industrial funding chosen in stage one influence the stage-two consulting equilibrium. An increase in either G or F shifts the firm's best reply down, because it is willing to pay less for consulting. These increases shift the researcher's best reply left, because she is willing to consult less. When her best reply is positively sloped, consulting time must decrease, but the effect on the fee is ambiguous, depending upon the relative magnitudes of these shifts. When her best reply is negatively sloped, the ultimate changes in both consulting and the fee are ambiguous. However, when the equilibrium is locally stable, as shown in Figure 2, then both equilibrium values cannot increase, or even remain constant. Either consulting or the fee must decrease.

If there are no spillovers from the university, $\beta = 0$, then the firm's best reply does not shift in response to a change in G. In this case, an increase in G decreases consulting and increases the fee. The analysis for an increase in the university infrastructure K_I is the same as that for G.

Finally, an increase in the firm's infrastructure K_O shifts the firm's best reply down. Spillovers from consulting back to university research increase her productivity there, and so the opportunity cost of consulting. An increase in K_O therefore leads her to spend less time consulting for any fee, so her best reply shifts left. As stated in Theorem 2, the effect on the fee is ambiguous, but consulting decreases when her best reply is positively sloped. If there are no such spillovers, $\lambda = 0$, then her best reply does not shift, so the fee decreases and the effect on consulting depends on the slope of her best reply: it increases (decreases) if her best reply is negatively (positively) sloped. Again, because her best reply does not shift, this result does not depend on assuming her utility is separable.

3.2 Stage One Equilibrium

In the first stage, the government funding agency and the firm simultaneously choose funding levels for the researcher's university project. As assumed above, the firm allocates a fixed amount $B_f > 0$ to R&D, and does not make major adjustments until the next budget cycle. Similarly, it is realistic to assume that the research budget of the government funding agency is also fixed at the level $B_g > 0$ during this period. To determine subgame perfect equilibria, we assume these funding choices are made subject to equilibrium behavior in stage two embedded in the equilibrium functions $t^*(G, F)$ and $c^*(G, F)$. Substituting these into (3) and (4) gives the "reduced form" payoffs

$$P_g(G, F) = EU_g(G, F, t^*(G, F), c^*(G, F)) \tag{9}$$

and

$$P_f(G, F) = E\Pi(G, F, t^*(G, F), c^*(G, F)). \tag{10}$$

By construction, a Nash equilibrium (G^*, F^*) of the simultaneous-move game with these payoffs is a subgame perfect equilibrium of the two-stage funding game. Appendix A formally states and proves existence of an equilibrium in this game, (G^*, F^*).[15]

Maximization of (9) by choosing $G \in [0, B_g]$ implicitly defines a best reply function $\hat{G}(F)$, giving the level of government funding for university research that maximizes the agency's expected utility for any choice of funding F by the firm. Similarly, maximization of (10)

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[15] Again, these equilibrium values are also functions of all the parameters of the model (α,β,q,x_I,K_I,x_O,K_O,S,L, γ).

by choosing $F \in [0, B_f]$ implicitly defines a best reply function $\hat{F}(G)$, giving the level of industrial funding for university research that maximizes the firm's expected profit for any funding level chosen by the government agency.[16] Again, however, because we are interested in deriving empirical implications, we focus on the interior equilibrium.

If the Nash equilibrium is interior, $G^* \in (0, B_g)$ and $F^* \in (0, B_f)$, then it must satisfy

$$\frac{\partial P_g(G^*, F^*)}{\partial G} = 0, \tag{11a}$$

and

$$\frac{\partial P_f(G^*, F^*)}{\partial F} = 0 \tag{11b}$$

where

$$\frac{\partial P_g(G, F)}{\partial G} = \left(\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G}\right)[U_g(R_{gs}) - U_g(R_{gf})] - V'(G) \tag{12a}$$

and

$$\frac{\partial P_f(G, F)}{\partial F} = \left(\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial F}\right)(\pi_I - L) + \left(\frac{\partial p_O}{\partial \tau}\frac{\partial t^*}{\partial F}\right)\pi. \tag{12b}$$

These conditions show both the initial marginal trade-offs between the benefits and costs of funding, and the effects of initial funding choices on the second stage equilibrium values. Notice that the agency's payoff does not depend on c^*, and firm's second stage optimal choice of c^* eliminates its effect on the first stage funding choice (via a standard envelope theorem application). An example of this equilibrium is depicted in Figure 3.

Conditions $(11a)$ and $(12a)$ show that increases in government funding directly increase effective funding e_I, and thus both the probability of success and expected utility, so the agency increases G until this marginal increase in expected utility from this project is offset by the marginal cost of reduced funding to other projects (embedded in V). Note that $\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G} > 0$ if $\frac{\partial t^*}{\partial G} < 0$, in which case it follows from $(12a)$ that the agency's best reply is interior as long as the opportunity cost of funding our heroine is not too high. The conditions in $(11b)$ and $(12b)$ show that devoting more funds to our heroines's university research has conflicting effects for the firm. First, it increases the probability of success in university research, and expected licensing profit. However, if $\frac{\partial t^*}{\partial F} < 0$, this reduces time in consulting, and therefore the probability of success in and expected profit from the firm's project. In this case, the firm funds university research as long as the increase in expected profit from licensing a success from the university project outweighs the expected profit loss from its own project.

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[16] We omit the parameters of the model as explicit arguments in these best reply functions for clarity of exposition.

Given the general ambiguity of $\frac{\partial t^*}{\partial G}$ and $\frac{\partial t^*}{\partial F}$, it is difficult to obtain comparative statics results on equilibrium levels of funding for university research. Indeed, even the slopes of the government and firm best reply functions are not obvious. Nevertheless, we can obtain results under reasonable assumptions. To do so we assume that

$$-\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_I}{\partial \tau \partial e_I} > 0 \text{ for } j = G,F \tag{13a}$$

and

$$\frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial j} > 0 \text{ for } j = G,F. \tag{13b}$$

These conditions essentially state that a stage-one increase in one type of external funding increases the marginal effect of the other type of external funding on the stage-two equilibrium probability of success in university research, in which case we can show the following.

Theorem 4 *Assume that equilibrium consulting time is decreasing in firm funding,* $\frac{\partial t^*}{\partial F} < 0$*, that* (13) *holds, and that second-order effects on equilibrium consulting times are negligible,* $\frac{\partial^2 t^*}{\partial i \partial j} \approx 0$*, for all parameters* i *and* j*. Then:*
(i) The first-stage best reply function of the funding agency is positively sloped.
(ii) The first-stage best reply function of the firm is positively sloped if, in addition, an increase in government funding decreases equilibrium consulting time, $\frac{\partial t^*}{\partial G} < 0$*, and sufficiently decreases equilibrium effective funding for the firm's consulting project,* $\frac{\partial e_O^*}{\partial G} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial G})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$.

Two points should be noted. First, the conditions in (13) and a negative effect of firm funding on equilibrium time spent consulting are sufficient for the government's best reply function to be positively sloped. Second, the additional hypotheses in (ii) of this theorem states that, although there are spillovers from university research to the firm's project, they cannot outweigh the reduced time the researcher spends in consulting if G increases, so equilibrium effective funding e_O^* for the firm's project decreases. This guarantees that an increase in G also results in a decrease in the marginal effect of F on the stage-two equilibrium probability of success in the firm's consulting project.

When the best reply functions are positively sloped, as depicted in Figure 3, we can identify some comparative statics results for the first stage.

Theorem 5 *Under the hypotheses of Theorem 4, if the first-stage equilibrium is locally stable, then an increase in research funding within the university, license revenue, or her share of it must increase equilibrium university research funding from both the government and the firm (*$\frac{\partial G^*}{\partial j} > 0$ *and* $\frac{\partial F^*}{\partial j} > 0$ *for* $j = K_I,L,\gamma$*) if, in addition, this sufficiently decreases* *equilibrium effective funding for the firm's consulting project,* $\frac{\partial e_O^*}{\partial j} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial j})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$ *for* $j = K_I, L, \gamma$.

An increase in the level of university research support, license revenue, or her share of it shifts the agency's best reply up, which must increase funding for her university research from both external sources. In each of these cases, the additional hypothesis is sufficient, but not necessary, to guarantee that the firm's best reply shifts right. Because each of these changes has an ambiguous effect on total consulting expenditure, c^*t^*, the condition $\frac{\partial e_O^*}{\partial j} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial j})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$ states that this effect is either negative, or not too positive. All other changes are ambiguous in this case.

4 Econometric Analysis

In this Section we focus on empirical estimates of the two stages of the model. While the ideal data would include time spent in consulting and the fee paid, to our knowledge such data are not available. However, because our interest is in consulting that is essentially research in firm labs, we are able to exploit a unique data set of 1679 patents on which 458 faculty from eight major US universities are listed as inventors. A patent lists, not only the inventors, but also the patent's assignee(s) (that is the patent's owner(s)). By institutional policies, both firms and universities typically claim ownership of inventions resulting from projects in their labs.[17] In our data, thirty percent of the patents are assigned to firms. In a series of interviews with faculty, university licensing professionals, and firm R&D executives, we were told that such assignments are almost exclusively the result of faculty consulting in firm labs.[18] Thus we take firm assignment as our proxy for consulting. While this measure ignores consulting that does not lead to patents, it clearly indicates inventorship on firm projects, such as those in our theory, which have scientific merit.

The eight universities in our sample are Cornell, Georgia Institute of Technology, Massachusetts Institute of Technology, Purdue, Stanford, Texas A & M, Pennsylvania, and Wisconsin. For each of the faculty at these universities in 1993, we have detailed annual information on faculty publications, citations and research funding. We restrict attention to faculty in years in which they applied for a patent that was granted between 1993 and 1999. This yields 1679 patent/inventor pairs where assignment of the patent is either to the university

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[17] For federally funded inventions, the Bayh-Dole Act allows universities to claim ownership and for non federally funded inventions, with rare exception universities claim ownership of inventions using their resources.

[18] It is also possible that firm assigned patents are on inventions that should legitimately be assigned to the university. See Thursby *et al.* (2009) for an empirical analysis of this issue.

or to a firm of which 1532 are unique patents.[19] Details on our selection process are provided in Appendix B. When there are two or more faculty inventors on a patent we randomly drop the duplicate patents so that we consider each patent once in our econometric analysis.

The reason for focusing on the years in which a faculty member is known to have applied for a patent is simple. The model we develop is for those faculty who can conceivably consult with industry. If a university inventor applies in some year for a patent that is subsequently granted, then clearly some of her work was deemed to be useful, and hence one can argue that she could have consulted in that year. To maintain comparability across the sample we also restrict non-consulting observations to those years in which there is an application for a university-assigned patent. Finally, we use patent characteristics as a measure of the focus of research. Restricting attention to years with patent applications excludes those faculty who consulted or could have consulted with industry on non-patentable projects, but, more to the point, it excludes faculty who were not of interest to industry as consultants.

It is also important to recognize that while the theoretical model yields a number of hypotheses that are testable in principle, a number of these concern the consulting fee c^* which we do not observe. Recall also that a number of results depend on the slope of the researcher's consulting supply function or the slopes of the funding best reply functions, and hence are not testable. Thus some of what follows is properly regarded as estimation of the system, rather than testing.

4.1 Consulting

For the second stage, we use a logit regression to explain the probability that a patent is assigned to a university, $P(UNIVASSGN_i = 0)$, rather than to a firm, $P(UNIVASSGN_i = 1)$,where i refers to a patent/inventor pair. Since assignment of a faculty patent to a firm is largely the outgrowth of consulting we interpret the probability of firm assignment as a measure of time spent in consulting t^*.

According to our theoretical model, regressors in the logit model should include measures of government and industry funding, G and F, the researcher's quality, q, the scientific merit of the university and firm projects, x_I and x_O, research support provided by the university and firm, K_I and K_0, the inventor's share γ of university license revenue L, and the two spillover parameters β and λ. Of these variables we have direct measures only for K_I, G, F,

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[19] A number of the firms in the sample are firms in which the inventor is a principal (founder, CEO and/or member of the scientific advisory board). However, our model does not differentiate between consulting with a start-up and consulting with an established firm thus our empirical analysis does not differentiate patents assigned to start-ups from other firm assigned patents.

Also an additional 80 patents were found but these were either unassigned or they had multiple assignees. These are not included in the analysis.

q, and γ.[20]

While the comparative statics for the consulting stage yield few testable hypotheses, three that we can test are (i) an increase in γ should decrease t^*, and if there are no spillovers, i.e., $\beta = 0$, then (ii) an increase in either government funding or K_I should also decrease t^*.

For each faculty member, we include the faculty member's yearly total U.S. government sponsored research funds and the total industry sponsored research funds received in the year prior to the patent application (LAG_GOV_FND and LAG_IND_FND) as measures of G and F. This information was provided by the respective university Office of Sponsored Programs. For multiple year and/or multiple principle investigator awards, we assume that expenditures were uniform across years and/or principle investigators.

For each university we also have information on γ, the inventor's share of university licensing income. If the university has a sliding scale we use the inventor share ($INVENTSHARE$) for income between \$25k and \$50k since the average licensing revenue for an active license in the US lies between those figures (AUTM, various years).

As proxies of faculty quality we use the number of publications by the faculty member in the year prior to the patent application (LAG_PUBS) and the total number of citations those publications received through 2003 (LAG_PUB_CITES). The data are from ISI Web of Science. While citations may indicate inventor quality, it is likely also that faculty who conduct more fundamental work are cited more (holding constant the number of publications). Thus LAG_PUB_CITES may reflect both inventor quality q and university project difficulty x_I.

We have four proxies for university research support K_I. First, we use the National Research Council's (1995) survey measure of the quality of the inventor's academic department ($DEPT\_QUAL$) where higher values are assigned to higher quality departments. High quality departments are generally considered to be relatively resource rich both in terms of research tools and the knowledge base of faculty colleagues.[21] Second, the average number of university-based co-authors per article ($AVER\_CO\_AUTHORS$) is a reflection of the size of the network of the faculty member's colleagues. The third measure is total university research expenditures, in current dollars ($UNIV\_RD\_EXPEND$). The final measure reflects university teaching loads, which, all else equal, should be negatively related to research. Instead of detailed data on teaching loads we have department level data on the ratio of full time graduate students to faculty in 1992 ($STUDENT\_FAC\_RATIO$) (National

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[20] The aggregate annual amount of licensing revenue by university is available. However, the appropriate value of L is the licensing revenue earned by the university for that specific project if it is licensed.

[21] If $DEPT\_QUAL$ and inventor quality q are positively correlated, then $DEPT\_QUAL$ can pick up some of the effects of q. In our data, however, our measures of inventor quality, numbers of publications and citations, have simple correlations of only 0.141 and 0.209, respectively, with the NRC ranking.

Research Council, 1995). Unfortunately, we do not have information on graduate students or faculty for other years, however, our prior is that these figures do not vary substantially year to year. $STUDENT\_FAC\_RATIO$ is a better measure of teaching responsibilities than the alternative university wide student/faculty ratio. This latter measure includes all university departments instead of just the department of the faculty member whose consulting we wise to examine. All else equal, the higher the graduate student/faculty ratio, the greater the teaching and advising roles of the faculty. All else, of course, is not equal. In particular, we need to control for the level of outside research funding of the department. We include two such controls. The first is the ratio of the number of department research grants to the number of faculty in 1992 (TOT_GRANTS_RATIO). The second is the ratio of the department's total research funding (industry and federal) to the number of faculty in 1992 (TOT_FUNDS_RATIO). TOT_GRANTS_RATIO captures, to some extent, the breadth of awards while TOT_FUNDS_RATIO captures the level of funding within the department. Both are measures in the year 1992 since the number of students is also for 1992. In summary, $DEPT\_QUAL$, $AVER\_CO\_AUTHORS$ and $UNIV\_RD\_EXPEND$ are each expected to be positively correlated with K_I while $STUDENT\_FAC\_RATIO$ is expected to be negatively correlated with K_I. We do not have a proxy for K_0.

To control for project difficulties x_I and x_O we use several measures of patent characteristics that reflect how important and/or fundamental is the patent. More important and/or fundamental patents are expected to emerge from more difficult (scientifically meritorious) problems. Three of the measures are backward looking. The first is the number of backward citations to prior patents ($PATENT\_CITES$) contained in the focal patent. The larger the number of backward citations the larger is the existing body of related patented work, so that we would expect patents with more backward citations to be more incremental and hence of less scientific merit. The second backward looking measure is the Trajtenberg *et al.* (1997) measure of patent originality ($ORIGINAL$). $ORIGINAL$ is based on a Herfindahl index that reflects the dispersion of citations made by the patent across patent classes. The originality score is higher the wider the range of classes to which the patent makes citations. A score of zero indicates that all citations to prior art are in a single patent class and scores close to one indicate citations to many classes. A patent is considered more original if it cites prior art from many rather than few technology classes. Both $PATENT\_CITES$ and $ORIGINAL$ are from the NBER Patent Database (Hall *et al.* 2001). We also include as an additional backward measure the number of non-patent publications cited as prior art in the patent ($ARTICLE\_CITES$). As a forward looking measure we include the number of forward citations (FOR_CITES) received by the patent by October 2006. It reflects importance of the patent in the sense that the patent has been considered prior art by either subsequent inventors or patent examiners. According to our theory, with the exception of $PATENT\_CITES$ the patent characteristics variables should increase the probability of assignment to the university; $PATENT\_CITES$ will decrease the probability.

Additional controls are indicator variables for major program field of the inventor: $PHYSCI = 1$ if the inventor is in the physical sciences and $ENG = 1$ for engineering faculty; the excluded category is biological sciences. When university fixed effects are not included we include an indicator variable for public versus private university ($PUBLIC = 1$ if the university is public) and an indicator variable for whether the university is located in an urban area ($URBAN = 1$ if the university is located in an urban area). Thursby *et al.* (2009) suggest that urban areas might provide more opportunities for consulting. Public universities often are expected to interact with (particularly local) firms to meet economic development goals (Thursby *et al.* 2009, Belonzon and Schankerman 2009).

Finally, controls are included for the inventor's gender ($MALE = 1$ if the inventor is male) and age at the time of the patent assignment (AGE). Thursby and Thursby (2007) find significant gender differences in faculty propensity to engage in licensing activities and Azoulay *et al.* (2007) find significant gender effects on faculty patent activity. In our theoretical model we show a relationship between inventor assets and consulting. To the extent that assets rise with age, we expect age and assets to be positively related, so age may be a proxy for assets. Thursby *et al.* (2007) argue that age effects on faculty commercialization activities are non-linear, so we also include the square of age ($AGESQ$). In their theoretical model Thursby *et al.* (2007) show that tenure can have a dramatic effect on faculty licensing. Unfortunately, we do not know for certain if or when a faculty member obtains tenure, but we do know the start date at their university. In the event that the "tenure clock" started when they were first employed at this university we can measure tenure as starting in the 7th year of their employment. $TENURE = 1$ indicates that the faculty member has tenure according to our algorithm. Our measure of tenure provides an undercount. Our earliest observation is for 1989 and the latest is for 1999 with over 92% occurring between 1994 and 1997. To account for any possible change over time in the propensity to consult we include a time trend ($TREND$) equal to 1 for the years 1989-91 and equal to 8 for the year 1998-99. The early and late years are aggregated because of few observations in the early and late years.

One hundred and fifty-eight of the patents have more than one faculty inventor. We randomly drop the duplicates so that each patent appears only once in the data. Summary statistics are found in Table 1 and the logit results in terms of odds ratios are given in Table 2. In our econometric analysis we use logs of LAG_GOVFND, LAG_INDFND, LAG_PUBS, LAG_PUB_CITES, $PATENT\_CITES$, $ARTICLE\_CITES$, FOR_CITES,

$UNIV\_RD\_EXPEND$ and $AVER\_CO\_AUTHORS$. Faculty can appear multiple times in the data so that cluster standard errors are used to account for potential non-independence of observations when faculty appear multiple times in the data. In Part A are results for university fixed effects. In Part B we include $PUBLIC$, $INVENTSHARE$ and $URBAN$; variables which cannot be used in the fixed effects estimation. Part A is our preferred specification since unobserved heterogeneity across universities could lead to bias in all estimated coefficients in the Part B regression.

Recall from our theoretical analysis that the impact of government funding on consulting should depend on the existence of spillovers. When $\beta > 0$, an increase in government funding G or an increase in university-provided research support K_I shifts the researcher's best reply back and the firm's best reply down, hence the ambiguous theoretical results. When $\beta = 0$, however, only the researcher's best reply shifts with an increase in government funding or university research support, implying a decrease in consulting. Therefore, a positive empirical relationship between G and consulting or K_I and consulting is possible only if $\beta > 0$. When significantly different from zero our measures show that higher levels of G and K_I are positively associated with firm assignment. That is, LAG_GOVFND has an odds ratio greater than one as does $AVER\_CO\_AUTHORS$. $STUDENT\_FAC\_RATIO$, which is negatively related to K_I, has an odds ratio less than one (though it is significant only in Part A). Thus our results clearly imply positive spillovers from the university to the firm. Further, recall from Theorem 2 that with a spillover, an increase in government funding will increase consulting only when the researcher's best reply function is negatively sloped in equilibrium. With a negatively sloped function, the theory also predicts the negative impact of quality on consulting. In our empirical results lagged publications decrease the probability of firm assignment, but the coefficients are not significantly different from zero.

To the extent that the peer review process followed by federal agencies identifies the best researchers, one might argue that government funding is another measure of researcher quality. However, the effect of additional government funding is opposite that of publications and industry funding, and the latter variables are clearly measures of researcher quality. Thus, in these data government funding is a measure of something other than faculty quality.

From the theory, an increase in $INVENTSHARE$ should decrease consulting regardless of the researcher's best reply so that the coefficient of $INVENTSHARE$ should be negative. The correct sign is observed (see Part B), but it is not significant at conventional levels. However, with only 8 universities in the sample there is little variation in $INVENTSHARE$.

The coefficients of AGE and $AGESQ$ are neither individually nor jointly significantly different from zero in either regression.

The patent characteristic variables are consistent with our assumption that university research projects are more difficult, or fundamental, $x_I > x_0$. Specifically, the measure of patent originality ($ORIGINAL$) is associated with a lower probability of firm assignment and it is significant at the 1% level. The larger the number of backward patent citations ($PATENT\_CITES$) the greater is the likelihood that the patent is assigned to a firm, thus the more incremental patents are assigned to the firm. FOR_CITES is not significant in either regression. Finally, the more citations to journal articles ($ARTICLE\_CITES$) the more likely it is that the patent is assigned to the university. This is the opposite of the effect of backward patent citations, and, while it might contradict the claim that firm patents are more incremental, it is likely only a sign that university inventions are closer to the academic literature, and thus more basic, than are firm inventions.

A number of robustness checks were considered; for parsimony, the detailed results are not presented. We included as a regressor the "expected" number of citations for a researcher's publications which is computed as the average number of citations received by articles in the journals and years in which the researcher's publications appear. Expected citations are not significant and other results are unchanged. We also considered current year publications, publication citations, and government and industry funding rather than their lags. The only change of note is that government funding becomes insignificant in both models.

4.2 Government and Industry Funding

In our theoretical model, government and industry research funding are simultaneously determined, so the funding stage regressions explain both the amount of government research funding, GOV_FND, and industry research funds, IND_FND, received by an inventor in a year in which they applied for a patent. In several of our specifications we also include the lagged value of the dependent variable (LAG_GOV_FND or LAG_IND_FND).

As in the consulting stage, we use LAG_PUBS and LAG_PUB_CITES as measures of inventor quality. Lagged rather than current publications and citations are used to allow for a lag between funding applications and awards.[22] Lagged publications and citations most likely reflect the researcher's productivity at the time the funding was applied for. While it is standard to consider citations as a measure of quality, *a priori*, their effect on funding levels is not entirely clear because, as we noted above, it is likely that more highly cited faculty conduct more fundamental research. This is in addition to the fact that the comparative static effects of quality on funding are ambiguous. Nonetheless, it is important to control for measures of quality.

Also, as in the consulting stage, when university fixed effects are not included we include

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[22] We also considered regressions including one year lags of publications and citations as well as two year lags. Results are unchanged so we do not present them for the sake of parsimony.

the percentage of licensing revenue the university awards the inventor ($INVENTSHARE$) as well as whether the university is public ($PUBLIC$) and whether it is in an urban area ($URBAN$). Our preferred specification is the one that has university fixed effects.

The variable $SPILL$ is the number of the inventor's articles cited as prior art in the patent. We view this as a measure of the spillover from the inventor's university research. If we assume that the researcher's journal publications result primarily from solving her university problem, then the larger is $SPILL$ the more the firm-assigned patent relies on the inventor's university research. Given that many of the inventors in our sample have multiple patents in a year, we randomly select one of the patents to measure the level of spillover. Note that this measure of β could not be used in the consulting regression because $SPILL$ is always zero when $UNIVASSGN = 1$.

For K_I we continue to use the three variables $DEPT\_QUAL$, $AVER\_CO\_AUTHORS$, and $UNIV\_RD\_EXPEND$ which are expected to be positively associated with K_I and $STUDENT\_FAC\_RATIO$ is expected to be negatively associated with K_I (along with the controls TOT_GRANTS_RATIO and TOT_FUNDS_RATIO).

To control for funding differences across fields we include indicator variables for the major program area of the inventor ($PHYSCI$ and ENG).

To account for the many zero dependent variable observations we use Tobit models. Because our theoretical model assumes government and industry research funding levels are simultaneously determined, we use an instrumental variables estimator. Instruments for the endogenous funding levels are their lagged values. Results are found in Tables 3 and 4. Parts A and B of each table are specifications including lagged values of the dependent variable. In Parts C and D the lagged dependent variable is omitted.

The results for both funding equations suggest that, as in Theorem 4, government and industry funding within the university are strategic complements. In Table 3, the government funding regressions, the estimated coefficients of industry funding ($INDFND$) are always positive and, with one exception, are significantly different from zero. In Table 4, the industry funding regressions, the coefficient of government funding ($GOVFND$) is also positive in each of the specifications though it is significantly different from zero only in Parts C and D. All variables are measured in logs so that the estimated coefficients are elasticities and these estimated elasticities are very small. In other work (Thursby and Thursby 2009a) we found similar results in a larger dataset that is not conditioned on patenting. In that work the positive elasticities are always significantly different from zero and are of a similar size as found in Tables 3 and 4. Recall that in each equation we control for quality of the individual faculty member using their publications and citations. In each specification, the coefficient of publications is positive and generally significant. Thus, to the extent that these measures adequately capture quality, the complementarity of government and industry research funding is not driven by quality.

According to Theorem 5, when government and firm funding are strategic complements, an increase in university research support K_I should increase equilibrium funding from both the government and the firm. When the coefficients for our measures of K_I are significantly different from zero, they have the correct signs in Tables 3 and 4, with the exception of $STUDENT\_FAC\_RATIO$ in the industry funding specifications which has a positive and significant effect in two specifications. While, in general, we think of a higher student faculty ratio as a drain on research, in the case of industry funding one could argue that firms fund university research, among other reasons, to gain access to potential hires, suggesting a positive sign.

Also according to Theorem 5, an increase in γ should increase both types of funding in equilibrium. In each specification the coefficient of $INVENTSHARE$ is negative, though it is significantly different from zero (10% level) in only one specification. As noted above, our preferred specifications are those that include university fixed effects. It is possible that there is unaccounted university heterogeneity in the non fixed effects regressions.

Finally, the comparative static effects of all other parameters in the funding stage of the theoretical model are ambiguous. Empirically, $SPILL$ is our measure of the university-industry spillover β. While it is not significant in the government funding equation, it is always negative and significant in the industry funding regressions. Intuitively, this suggests that in their funding decisions, firms realize the potential for free riding. AGE and $AGESQ$ are included as measures of life cycle effects. Within the context of the theory, they should reflect assets, which are generally larger, for older faculty. We find strong evidence that both types of funding are increasing in AGE but at a decreasing rate.

5 Concluding Remarks

Despite survey results showing that industrial managers often consider consulting to be one of the more important mechanisms for industry to access university research, there is little research either theoretically or empirically of this mechanism. In this paper, we examine industrial consulting by university faculty as a mechanism for spillovers and do so in a context that allows us to link them to government, industry, and university funding decisions. We develop a theoretical model which yields predictions for the time spent consulting, the associated fee, and the level of government and industry support for university research as functions of faculty quality, project characteristics, the researcher's share of license revenue from the university project, R&D spillovers, university support for the researcher's internal project, as well as the willingness of the firm and government to sponsor the faculty member's research within the university. We then exploit a unique database of funding, publications, and patents for 458 faculty inventors to estimate parameters of the model.

Both the theory and empirics provide clear evidence of spillovers from government-supported university research to industry. Moreover, together they suggest an important role for university policies in influencing faculty research and consulting. In the consulting stage, we find that an increase in either license revenue or the share that accrues to the faculty will decrease consulting. Thus as long as the university project is more basic than the firm's, then contrary to the policy concern that licensing might reduce basic research, the increase in share increases the time devoted to basic research. Perhaps our most striking results regard the ability of university policies to leverage both government and industry research funding. We find evidence that government and industry funding for university research are strategic complements, in which case increases in the share of revenue universities allocate to their researchers and the university's research infrastructure increase both government and firm funding in the university.

We also find strong evidence of spillovers from the university to the firm. In the consulting stage, we find that consulting is positively associated with government funding. In the context of our theoretical model, this result is possible only if there is a spillover from the faculty researcher's government sponsored research to the firm's research problem.

Several qualifiers to our work suggest directions for further research. First, one fourth of the patents in the sample assigned to for-profit firms are assignments to firms in which the inventor is a principal (founder, CEO, and/or scientific advisor). A role as scientific advisor is consistent with our interpretation of the faculty researcher choosing $t > 0$ and is consistent with most university policies as long at $t \leq M$. The patent may or may not be a follow-on patent to one from the faculty researcher's university research, in which case we would interpret the follow on project as x_o. Moreover, most conflict of interest policies prohibit faculty from receiving sponsored research from their start ups, so that this example would be the special case of our model in which $F = 0$. Of course, we do not differentiate between start ups and other types of firms in the analysis so we abstract from many of the nuances of faculty start ups.

Second, our empirical measure of consulting is limited in that is only includes consulting that leads to patentable results, and we do not have measures of the reverse spillovers to the firm. Nonetheless, to our knowledge there do not exist available databases, the development of which would be a nontrivial task.

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A Appendix (Not for Publication)

A.1 Existence in the Second-Stage Game

Assume the firm allocates a fixed amount $B_f > 0$ to R&D in a given period, so $c \in [0, B_f/M]$.

Lemma 1 *Consider the strategic form game with the researcher and firm as the players, whose strategies are* $t \in [0, M]$ *and* $c \in [0, B_f/M]$*, and payoff functions are defined by (2) and (4). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium* $(t^*(G,F), c^*(G,F))$*.*

Proof. Because the number of players is finite, their strategy sets are compact and nonempty, and their payoff functions are continuous and strictly quasi-concave, this follows directly from the well-known existence theorem for strategic form games with continuous strategy spaces (see, for example, Friedman 1977).

A.2 Properties of the Researcher's Best Reply

Her best reply function, $\hat{t}(c)$, is defined by $\arg\max\{t \in [0, M] : EU(G, F, t, c)\}$. We suppress its dependence on the other parameters $j = G$,F,β,λ,q,x_I,K_I,x_O,K_O,A,L,γ for notational convenience. In general, we are interested in whether and when her best reply is positive, so she supplies time to consulting, whether and when she supplies all available time M to consulting, and the slope of her best reply. First note that, given the strict concavity of $EU(G, F, t, c)$ in t,

$$\begin{array}{rcll} & = & 0 & if \ \dfrac{\partial EU(G,F,0,c)}{\partial t} \leq 0 \\ \hat{t}(c) & \in & (0, M) & if \ \dfrac{\partial EU(G,F,0,c)}{\partial t} > 0 > \dfrac{\partial EU(G,F,M,c)}{\partial t} \\ & = & M & if \ \dfrac{\partial EU(G,F,M,c)}{\partial t} \geq 0 \end{array}$$

defines the range of her best reply.

Proof of Theorem 1. Note that when her best reply is interior, its slope is $\hat{t}'(c) = -(\frac{\partial^2 EU}{\partial t \partial c})/(\frac{\partial^2 EU}{\partial t^2})$, which has the sign of $\frac{\partial^2 EU}{\partial t \partial c}$ by the strict concavity of EU. From (8), with risk-neutrality, $\frac{\partial^2 EU}{\partial t \partial c} = [p_I - t\frac{\partial p_I}{\partial \tau}]\frac{\partial U(R_s,W_s)}{\partial Y} + [1 - p_I + t\frac{\partial p_I}{\partial \tau}]\frac{\partial U(R_f,W_f)}{\partial Y} > 0$ because $\frac{t}{p_I}\frac{\partial p_I}{\partial \tau} \geq 1$

implies $p_I \geq t\frac{\partial p_I}{\partial \tau}$, and $\frac{\partial U}{\partial Y} > 0$, $\frac{\partial p_I}{\partial \tau} > 0$, and $p_I < 1$ by assumption. However, if $\frac{t}{p_I}\frac{\partial p_I}{\partial \tau} < 1$, then $p_I < t\frac{\partial p_I}{\partial \tau}$ and the first term in $\frac{\partial^2 EU}{\partial t \partial c}$ is negative. Similarly, if she is risk averse, then $ct[p_I\frac{\partial^2 U(R_s,W_s)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_f,W_f)}{\partial Y^2}] < 0$ as well, so (8) is more likely to be negative. ■

Lemma 2 *The researcher's best reply function in consulting time, $\hat{t}(c)$, is positive only if $c_m = \min\{c : \frac{\partial EU(G,F,0,c)}{\partial t} = 0\}$ exists and is finite. If so, then $c_m > 0$, and her best reply is positive and increasing in the consulting fee, $\hat{t}(c) > 0$ and $\hat{t}'(c) > 0$, for all fees in a neighborhood above c_m.*

Proof. First observe from (6a) that $\frac{\partial EU(G,F,t,0)}{\partial t} = -\frac{\partial p_I}{\partial \tau}[U(R_s, A+\gamma L) - U(R_f, A)] < 0$ for all t because $R_s > R_f$, $\gamma L > 0$, and positive marginal utility imply that $U(R_s, A+\gamma L) > U(R_f, A)$, and $\frac{\partial p_I}{\partial \tau} > 0$. Hence, because t is constrained to be nonnegative, $\hat{t}(0) = 0$. That is, if we plotted $\frac{\partial EU(G,F,t,c)}{\partial t}$ as a function of c for fixed (G, F, t), then it would intersect the (vertical) utility axis at a negative value. Because $\frac{\partial^2 EU(G,F,t,c)}{\partial t^2} < 0$, if $\frac{\partial EU(G,F,0,c)}{\partial t} < 0$ for all $c \in [0, B_f/M]$, then $\frac{\partial EU(G,F,t,c)}{\partial t} < 0$ for all c, and consulting never occurs, $\hat{t}(c) = 0$ for all c. However, the slope of $\frac{\partial EU}{\partial t}$ with respect to c at $c = 0$ is $\frac{\partial^2 EU(G,F,0,c)}{\partial t \partial c} = p_I\frac{\partial U(R_s,W_s)}{\partial Y} + (1-p_I)\frac{\partial U(R_f,W_f)}{\partial Y} > 0$ Thus, it is possible that $\frac{\partial EU(G,F,t,c)}{\partial t}$ increases (though perhaps not monotonically) as c increases, and eventually intersects the (horizontal) c axis. If so, there exists a positive, finite c_m defined as above. By continuity, $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$. Note that c_m is the fee at which the function $EU(G,F,t,c)$ takes on its unconstrained maximum at $t = 0$ (or the smallest fee if this occurs for more than one value). Therefore, for all fees in a neighborhood above c_m, $EU(G,F,t,c)$ takes on its unconstrained maximum at some $t > 0$, so $\hat{t}(c) > 0$. Moreover, $\hat{t}'(c) > 0$ in this neighborhood from the proof of Theorem 1 because $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$. ■

Proof of Theorems 2 and 3. Using standard comparative statics, $\frac{\partial t^*}{\partial j} = [\frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial j} - \frac{\partial^2 E\Pi}{\partial c^2}\frac{\partial^2 EU}{\partial t \partial j}]/D_2$ and $\frac{\partial c^*}{\partial j} = [\frac{\partial^2 E\Pi}{\partial c \partial t}\frac{\partial^2 EU}{\partial t \partial j} - \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c \partial j}]/D_2$, for $j = G,F,\beta,\lambda,q,x_I, K_I,x_O,K_O,A,L,\gamma$ where $D_2 = \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c^2} - \frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial t} > 0$ by the assumption that the equilibrium is locally stable.

Differentiation yields $\frac{\partial^2 EU}{\partial t \partial G} = -\frac{\partial^2 p_I}{\partial \tau \partial e_I}[U(R_s,W_s) - U(R_f,W_f)] + c\frac{\partial p_I}{\partial e_I}[\frac{\partial U(R_s,W_s)}{\partial Y} - \frac{\partial U(R_f,W_f)}{\partial Y}] < 0$ if $\frac{\partial U(R_s,W_s)}{\partial Y} \leq \frac{\partial U(R_f,W_f)}{\partial Y}$, $\frac{\partial^2 EU}{\partial t \partial F} = \frac{\partial^2 EU}{\partial t \partial K_I} = \frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial K_O} = (\frac{\lambda}{K_O})\frac{\partial^2 EU}{\partial t \partial \lambda} = \lambda\frac{\partial^2 EU}{\partial t \partial G}$ (but $= 0$ if $\lambda = 0$ or $t = 0$), $\frac{\partial^2 EU}{\partial t \partial q} = -\frac{\partial^2 p_I}{\partial \tau \partial q}[U(R_s,W_s) - U(R_f,W_f)] + c\frac{\partial p_I}{\partial q}[\frac{\partial U(R_s,W_s)}{\partial Y} - \frac{\partial U(R_f,W_f)}{\partial Y}] < 0$ if $\frac{\partial U(R_s,W_s)}{\partial Y} \leq \frac{\partial U(R_f,W_f)}{\partial Y}$, $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = 0$, $\frac{\partial^2 EU}{\partial t \partial L} = [-\frac{\partial p_I}{\partial \tau}\frac{\partial U(R_s,W_s)}{\partial Y} + p_I c\frac{\partial^2 U(R_s,W_s)}{\partial Y^2}]\gamma = (\frac{L}{\gamma})\frac{\partial^2 EU}{\partial t \partial \gamma} < 0$, $\frac{\partial^2 EU}{\partial t \partial A} = -\frac{\partial p_I}{\partial \tau}[\frac{\partial U(R_s,W_s)}{\partial Y} - \frac{\partial U(R_f,W_f)}{\partial Y}] + c[p_I\frac{\partial^2 U(R_s,W_s)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_s,W_s)}{\partial Y^2}]$, and $\frac{\partial^2 EU}{\partial t \partial x_I} = -\frac{\partial^2 p_I}{\partial \tau \partial x_I}[U(R_s,W_s) - U(R_f,W_f)] + c\frac{\partial p_I}{\partial x_I}[\frac{\partial U(R_s,W_s)}{\partial Y} - \frac{\partial U(R_f,W_f)}{\partial Y}] - \frac{\partial p_I}{\partial \tau}\frac{\partial U(R_s,W_s)}{\partial R}R_s' + cp_I\frac{\partial^2 U(R_s,W_s)}{\partial Y \partial R}R_s'$.Next note that $\frac{\partial^2 E\Pi}{\partial c \partial t} = [\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2}c]t\pi < 0$ at an interior solution to the firm's problem, so (7) implies the consulting demand function is negatively sloped. Differentiation yields $\frac{\partial^2 E\Pi}{\partial c \partial G} = \frac{\partial^2 E\Pi}{\partial c \partial K_I} = \frac{\partial^2 p_O}{\partial e_O^2}\beta t\pi < 0$ if $\beta > 0$ (but $= 0$ if $\beta = 0$ or if $t = 0$),

$\frac{\partial^2 E\Pi}{\partial c \partial F} = \frac{\partial^2 p_O}{\partial e_O^2} t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} = \frac{\partial^2 p_O}{\partial e_O^2}(K_I + G)t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_I} = \frac{\partial^2 E\Pi}{\partial t \partial A} = \frac{\partial^2 E\Pi}{\partial c \partial L} = \frac{\partial^2 E\Pi}{\partial c \partial \gamma} = \frac{\partial^2 E\Pi}{\partial c \partial \lambda} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial q} = \frac{\partial^2 p_O}{\partial e_O \partial q} t\pi > 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} = \frac{\partial^2 p_O}{\partial e_O \partial x_O} t\pi < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} = \frac{\partial^2 p_O}{\partial e_O^2} t\pi < 0$. The assumption that $U(R, W) = f(R) + g(W)$ implies $\frac{\partial U(R_s,W_s)}{\partial Y} - \frac{\partial U(R_f,W_f)}{\partial Y} = 0$. Combining this and the preceding results proves all statements in Theorem 3 and all in Theorem 2 except for those about G, F, and K_I in (iv).

Finally, recall that $\frac{\partial^2 EU}{\partial t \partial j} < 0$ for $j = G, F, K_I$ if $\frac{\partial U(R_s,W_s)}{\partial Y} \leq \frac{\partial U(R_f,W_f)}{\partial Y}$, $\frac{\partial^2 E\Pi}{\partial c \partial F} < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial j} < 0$ for $j = G, K_I$ if $\beta > 0$. Therefore, under these conditions, $\frac{\partial t^*}{\partial j} + \frac{\partial c^*}{\partial j} = \{\frac{\partial^2 EU}{\partial t \partial j}[\frac{\partial^2 E\Pi}{\partial c \partial t} - \frac{\partial^2 E\Pi}{\partial c^2}] + [\frac{\partial^2 EU}{\partial t \partial c} - \frac{\partial^2 EU}{\partial t^2}]\frac{\partial^2 E\Pi}{\partial c \partial j}\}/D_2 > 0$ for $j = G, F, K_I$ if and only if both $\frac{\partial^2 E\Pi}{\partial c \partial t} - \frac{\partial^2 E\Pi}{\partial c^2} < 0$ and $\frac{\partial^2 EU}{\partial t \partial c} - \frac{\partial^2 EU}{\partial t^2} < 0$, which contradicts the local stability condition. This proves statements (iv)(b) and (iv)(b) in Theorem 2. ■

A.3 Existence in First-Stage Game

Lemma 3 *Consider the strategic form game with the government funding agency and firm as the players, whose strategies are $G \in [0, B_g]$ and $F \in [0, B_f]$, and payoff functions are defined by (8) and (9). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium (G^*, F^*), and $(G^*, F^*, t^*(G^*, F^*), c^*(G^*, F^*))$ is the subgame perfect equilibrium of the two-stage funding game.*

A.4 Properties of First-Stage Equilibrium

Proof of Theorem 4. V. Proof of Theorem 4. Set $\Theta = -\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial \tau \partial e_I}$, $\Lambda = \frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial G}$, $\Phi = -\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial F} + \frac{\partial^2 p_I}{\partial \tau \partial e_I}$, $\Omega = \frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial F}$, and $\Delta U_g = U_g(R_{gs}) - U_g(R_{gf}) > 0$. Recall from (12) $\Theta > 0$, $\Lambda > 0$, $\Phi > 0$, and $\Omega > 0$ by assumption. Then $\frac{\partial^2 P_g}{\partial G \partial F} = [\Theta(-\frac{\partial t^*}{\partial F}) + \frac{\partial p_I}{\partial \tau}(-\frac{\partial^2 t^*}{\partial G \partial F}) + \Lambda]\Delta U_g > 0$ because $\frac{\partial t^*}{\partial F} < 0$ and $\frac{\partial^2 t^*}{\partial F \partial G} \approx 0$ by assumption. Similarly, $\frac{\partial^2 P_f}{\partial F \partial G} = [\Phi(-\frac{\partial t^*}{\partial G}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial F} + \Omega](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial G}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial G})]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial G}\}\pi > 0$ because $\frac{\partial t^*}{\partial G} < 0$, $\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial^2 t^*}{\partial F \partial G} \approx 0$, and $\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial G}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial G}) \leq 0$ by the hypothesis of the theorem. ■

Proof of Theorem 5. In this case, we have $\frac{\partial G^*}{\partial j} = [\frac{\partial^2 P_g}{\partial G \partial F}\frac{\partial^2 P_f}{\partial F \partial j} - \frac{\partial^2 P_f}{\partial F^2}\frac{\partial^2 P_g}{\partial G \partial j}]/D_1$ and $\frac{\partial F^*}{\partial j} = [\frac{\partial^2 P_f}{\partial F \partial G}\frac{\partial^2 P_g}{\partial G \partial j} - \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F \partial j}]/D_1$, for $j = \beta, \lambda, q, x_I, K_I, x_O, K_O, \gamma, A, L$, where $D_1 = \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F^2} - \frac{\partial^2 P_g}{\partial G \partial F}\frac{\partial^2 P_f}{\partial F \partial G} > 0$ by the assumption that the equilibrium is locally stable, and where $\frac{\partial^2 P_g}{\partial G \partial F} > 0$ and $\frac{\partial^2 P_f}{\partial F \partial G} > 0$ from the preceding theorem. First, observe that $\frac{\partial^2 P_g}{\partial G \partial j} = [\Theta(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial j}]\Delta U_g > 0$ for $j = \gamma, L$ because $\frac{\partial t^*}{\partial j} < 0$ for $j = \gamma, L$ and $\frac{\partial^2 t^*}{\partial i \partial j} \approx 0$. Next note that $\frac{\partial^2 EU}{\partial t \partial G} = \frac{\partial^2 EU}{\partial t \partial K_I}$ and $\frac{\partial^2 E\Pi}{\partial c \partial G} = \frac{\partial^2 E\Pi}{\partial c \partial K_I}$ imply that $\frac{\partial t^*}{\partial K_I} = \frac{\partial t^*}{\partial G} < 0$, so $\frac{\partial^2 P_g}{\partial G \partial K_I} = [\Theta(-\frac{\partial t^*}{\partial K_I}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial K_I} + \Lambda]\Delta U_g > 0$ as well. However, $\frac{\partial^2 P_g}{\partial G \partial j} = [\Theta(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial j}]\Delta U_g$ for

$j = \beta, x_O$, $\frac{\partial^2 P_g}{\partial G \partial K_O} = [\Theta(-\frac{\partial t^*}{\partial K_O}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial K_O} + \lambda\Lambda]\Delta U_g$, $\frac{\partial^2 P_g}{\partial G \partial \lambda} = [\Theta(-\frac{\partial t^*}{\partial \lambda}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial \lambda} + K_O\Lambda]\Delta U_g$, $\frac{\partial^2 P_g}{\partial G \partial q} = [\Theta(-\frac{\partial t^*}{\partial q}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial q} - \frac{\partial^2 p_I}{\partial \tau \partial q}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I \partial q}]\Delta U_g$, and $\frac{\partial^2 P_g}{\partial G \partial x_I} = [\Theta(-\frac{\partial t^*}{\partial x_I}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial x_I} - \frac{\partial^2 p_I}{\partial \tau \partial x_I}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I \partial x_I}]\Delta U_g + \Theta U_g' R_{sg}'$ are all ambiguous. Similarly, $\frac{\partial^2 P_f}{\partial F \partial j} = [\Phi(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial j}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}\}\pi > 0$ for $j = \gamma, L$ and $\frac{\partial^2 P_f}{\partial F \partial K_I} = [\Phi(-\frac{\partial t^*}{\partial K_I}) + \Omega - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial K_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial K_I} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial K_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial K_I}\}\pi > 0$ because $\frac{\partial t^*}{\partial j} < 0$ and $\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial j}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial j}) \leq 0$ for $j = \gamma, L, K_I$ by the hypothesis of the theorem. Next note that $\frac{\partial^2 P_f}{\partial F \partial \beta} = [\Phi(-\frac{\partial t^*}{\partial \beta}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \beta}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial \beta} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial \beta}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \beta}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial K_O} = [\Phi(-\frac{\partial t^*}{\partial K_O}) + \Omega\lambda - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial K_O}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial K_O} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial K_O}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial K_O}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial \lambda} = [\Phi(-\frac{\partial t^*}{\partial \lambda}) + \Omega K_O - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \lambda}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial \lambda} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial \lambda}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \lambda}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial q} = [\Phi(-\frac{\partial t^*}{\partial q}) + \frac{\partial^2 p_I}{\partial e_I \partial q} - \frac{\partial^2 p_I}{\partial \tau \partial q}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial q}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial q} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial q} + \frac{\partial^2 p_O}{\partial \tau \partial q}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial q}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial x_I} = [\Phi(-\frac{\partial t^*}{\partial x_I}) + \frac{\partial^2 p_I}{\partial e_I \partial x_I} - \frac{\partial^2 p_I}{\partial \tau \partial x_I}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_I} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}\}\pi$, and $\frac{\partial^2 P_f}{\partial F \partial x_O} = [\Phi(-\frac{\partial t^*}{\partial x_O}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial x_O}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}\}\pi$ are all ambiguous. The statements of the theorem then follow immediately from these results plus locally stability. ■

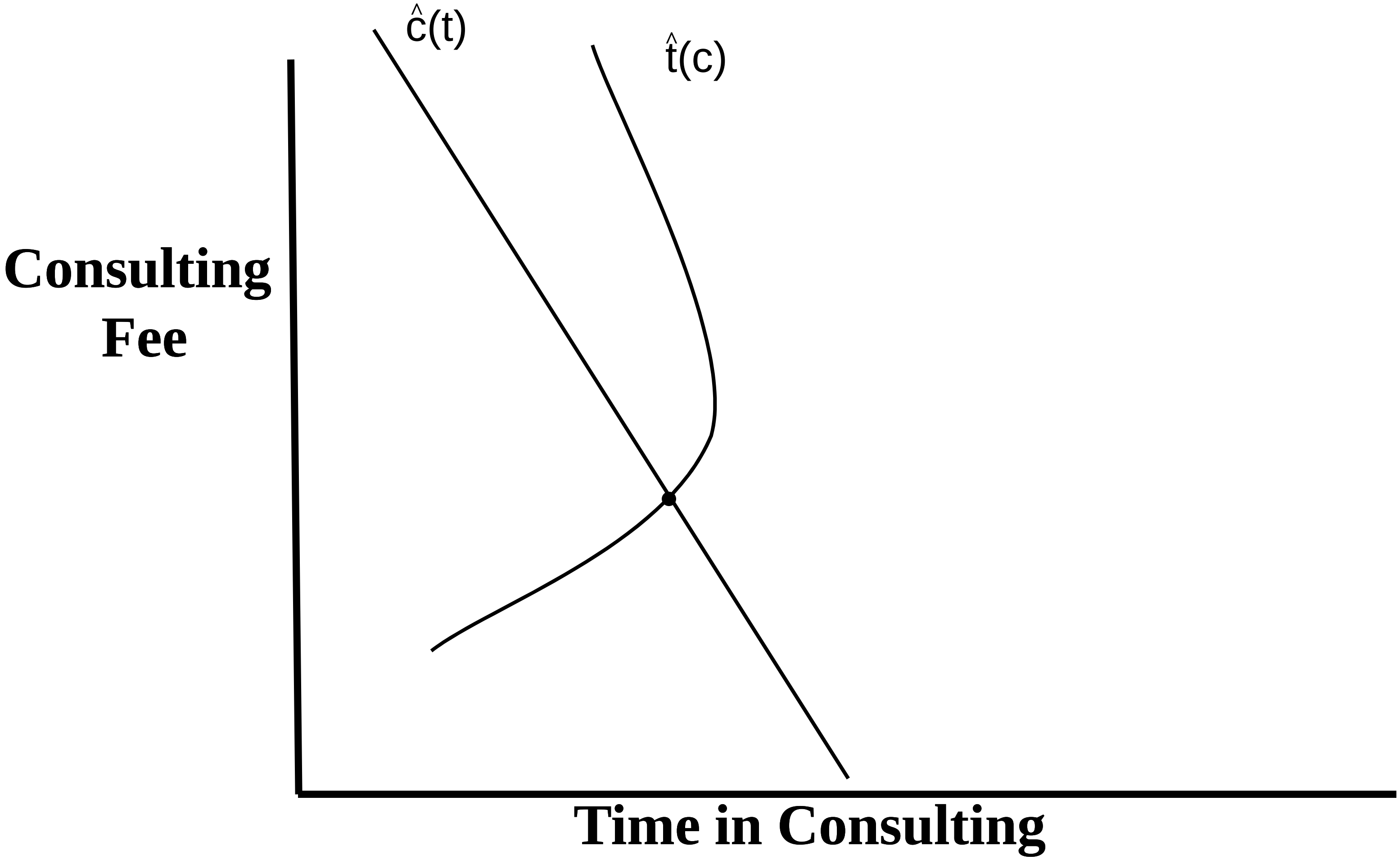
B Method of Identifying Faculty Patents

We started with the names of every scientist and engineer in PhD granting departments at Cornell, Georgia Institute of Technology, Massachusetts Institute of Technology, Purdue, Stanford, Texas A & M, Pennsylvania, and Wisconsin reported in the National Research Council's (1995) survey of PhD granting departments in the US. The list is for faculty in residence in 1993. The names were compared to inventor names on US patents granted between 1993 and 1999. Once names had been paired we used a multi-staged name screening process to insure that the faculty member and the inventor are the same. The first step was to check the distance between the zip code of the university and the zip code of the city of residence of the inventor. If the distance was more than 50 miles the patent was removed from the database. The average zip code distance in the final data is less than 7 miles.

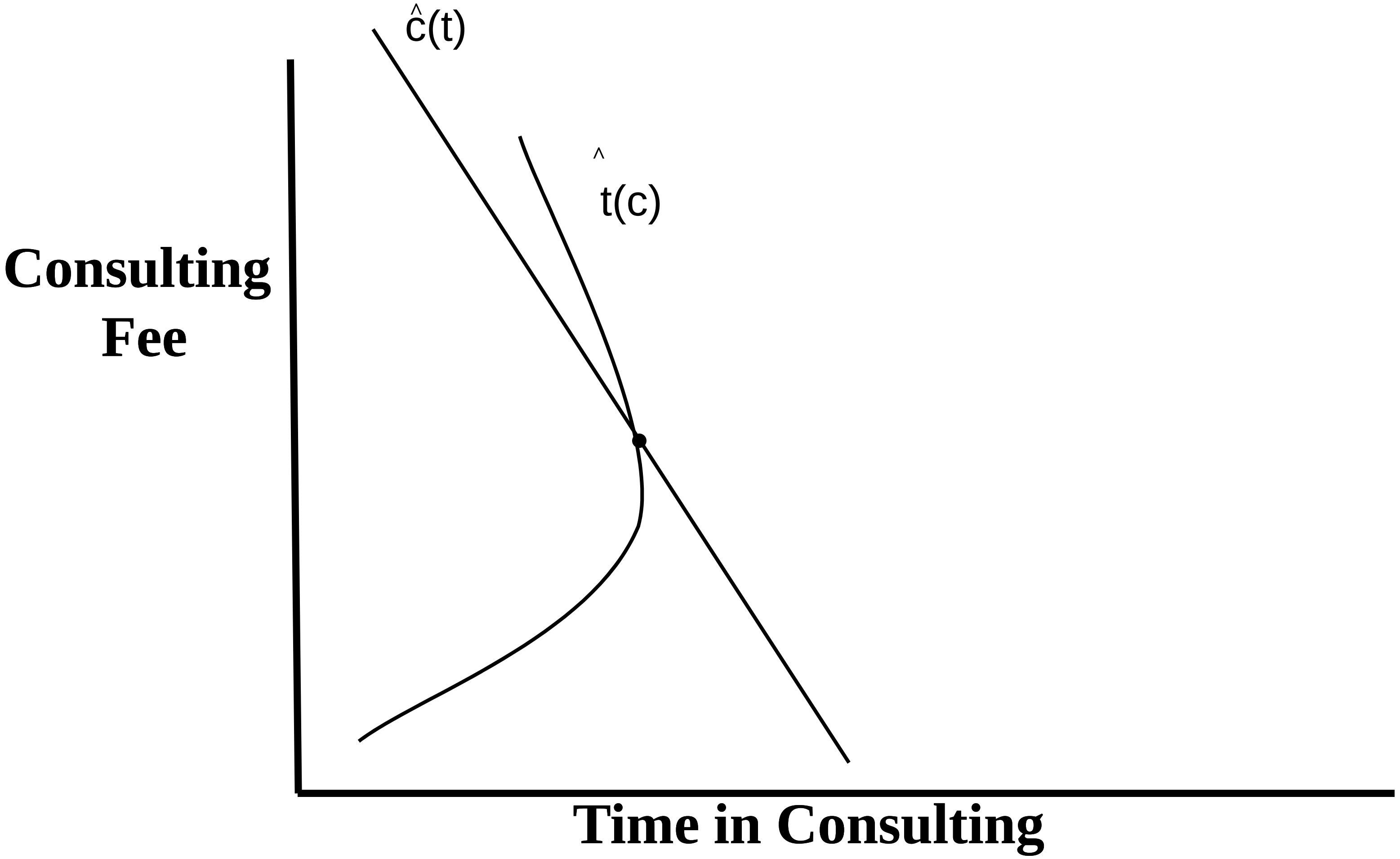
We then used the surname data from the 1990 Census to evaluate the incidence of each name. At this point we used various filters to eliminate common names. The filter was least dense when we had a match on first, middle and last name for the university faculty listing and the name on the patent. The filter was most dense when we could only match a last name and first initial. About 10% of the matches are eliminated because of common names.

From the NRC survey we know that a faculty member is at a respective university in 1993 (the year of the survey). Our procedure for verifying institutional affiliation for inventors on applications filed in years other than 1993 was the following. If the assignment was to the inventor's 1993 university we assumed the inventor was with the university. If the assignment was not to that university, we used a combination of web searches and the faculty listing in the National Faculty Directory to verify affiliation. If we determined that a faculty member was on the faculty in any year after 1993 we assumed she was on the faculty in the intervening years between 1993 and the latter year. If we could not verify faculty affiliation for the patent application year the patent was dropped.

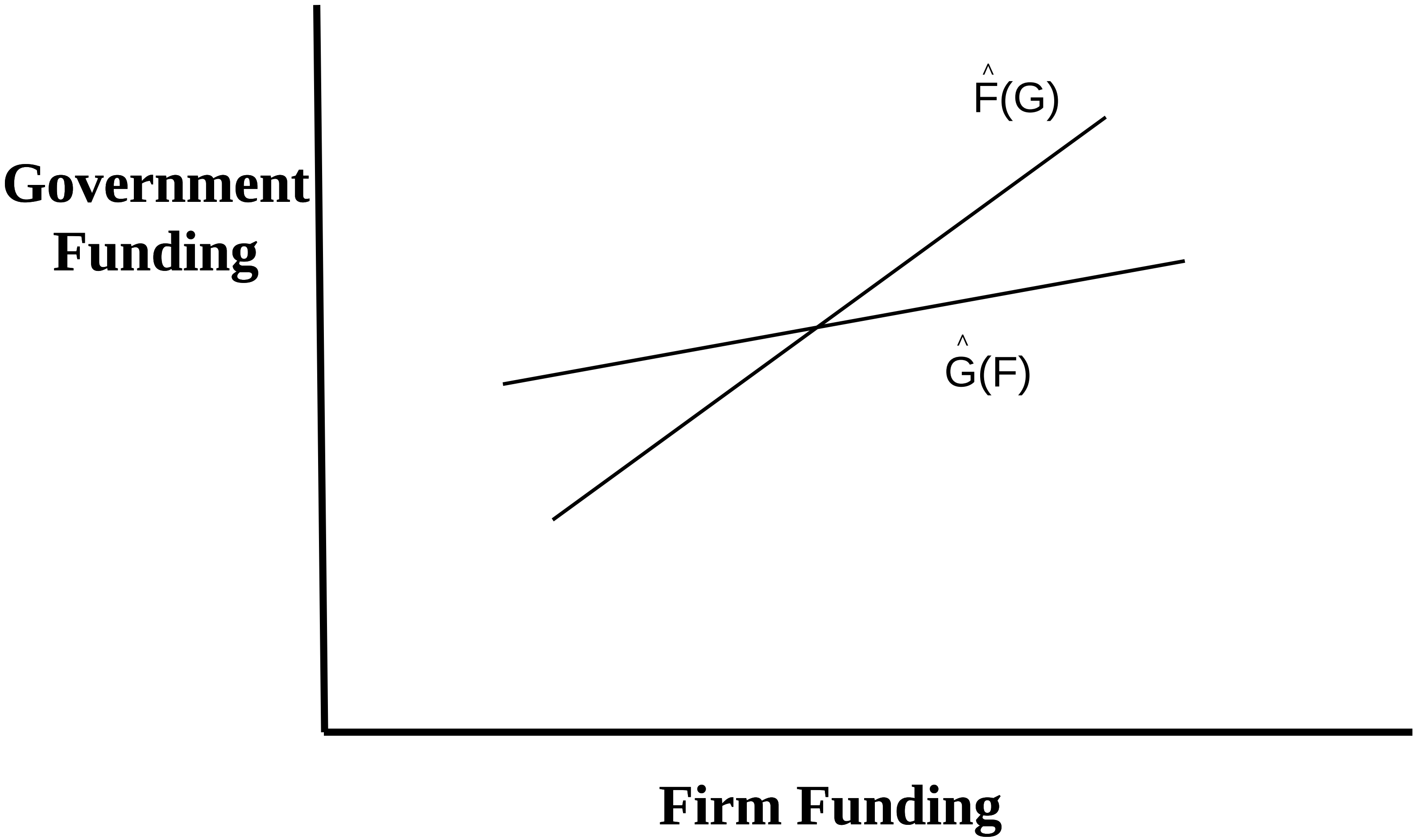
The final data contains 1690 patent/inventor pairs where assignment of the patent is either to the university or to a firm. These pairs include 1532 patents and 458 faculty inventors. When there are two or more faculty inventors on a patent we randomly drop the duplicate patents so that we consider each patent once in our econometric analysis.



Second Stage
Figure 1



Second Stage
Figure 2



First Stage
Figure 3

Table 1. Summary Statistics

		Obs.	Mean	St. Dev.	Min	Max
AGE	Inventor age	1455	49.318	10.009	28	83
ARTICLE_CITES	Articles cited in patent	1521	17.108	23.666	0	203
AVER_CO_AUTHORS	Average number of univ. co-authors	1521	12.532	43.210	0	418.843
BIOSCI	Bio science faculty	1521	0.342	0.474	0	1
DEPT_QUAL	Dept. quality	1521	4.277	0.530	2	4.97
ENG	Engineering faculty	1521	0.460	0.499	0	1
FOR_CITES	Forward patent citations	1521	21.131	36.994	0	459
GOV_FND	Federal funding	1521	0.838	1.841	0	15.021
IND_FND	Industrial funding	1521	0.165	0.562	0	4.177
INVENTSHARE	Inventor share of licensing revenue	1521	30.486	7.208	20	50
MALE	Inventor is male	1494	0.952	0.214	0	1
ORIGINAL	Measure of patent originality	1374	0.465	0.291	0	0.930
PAT_CITES	Patent citations to prior patents	1521	11.938	19.901	0	354
PERIOD	Yearly trend	1521	4.505	1.742	1	8
PHYSCI	Physical science faculty	1521	0.198	0.399	0	1
PUB_CITES	Citations to publications	1521	264.858	533.241	0	6557
PUBLIC	University is public	1521	0.318	0.466	0	1
PUBS	Annual publications	1521	7.300	8.525	0	51
STUDENT_FAC_RATIO	Graduate student faculty ratio	1441	3.655	2.320	0	13
TENURE	Facult member is tenured	1521	0.774	0.418	0	1
TOT_FUNDS_RATIO	Ratio of total dept. funding to faculty	1492	0.552	0.747	0	4.775
TOT_GRANTS_RATIO	Ratio of number of dept. grants to facutly	1521	3.640	1.782	0.302	10.333
UNIV_RD_EXPEND	University R&D expenditures	1521	362493	70680	161714	509782
UNIVASSIGN	Patent is assigned to university	1521	0.690	0.463	0	1
URBAN	University in urban area	1521	0.645	0.479	0	1

All funding variables are in millions of real dollars.

Table 2. Consulting

Dependent Variable: ASSIGN=1 if assigned to a firm.

	Part A		Part B	
	Odds Ratio	t-Statistic	Odds Ratio	t-Statistic
LAG_PUB	0.791	-1.01	0.739	-1.36
LAG_PUB_CITES	0.970	-0.35	1.003	0.04
LAG_GOVFND	1.021	2.01 **	1.023	2.13 **
LAG_INDFND	0.983	-1.23	0.982	-1.28
TENURE	1.088	0.27	0.990	-0.03
ENG	0.617	-0.92	0.562	-1.07
PHYSIC	1.632	0.91	1.489	0.72
ORIGINAL	0.313	-2.87 ***	0.303	-2.94 ***
PATENT_CITES	2.248	5.20 ***	2.301	5.29 ***
FOR_CITES	0.965	-0.35	0.987	-0.14
ARTICLE_CITES	0.704	-3.36 ***	0.702	-3.34 ***
AGE	0.826	-1.61	0.857	-1.29
AGESQ	1.002	1.58	1.001	1.26
MALE	0.738	-0.54	0.853	-0.31
TREND	1.093	1.38	1.096	1.6
DEPT_QUAL	1.001	0.00	1.582	1.44
UNIV_RD_EXPEND	0.353	-0.68	0.350	-1.41
AVER_CO_AUTHORS	1.336	2.98 ***	1.291	2.85 ***
STUDENT_FAC_RATIO	0.865	-1.66 *	0.948	-0.61
TOT_GRANTS_RATIO	1.023	0.18	0.989	-0.09
TOT_FUNDS_RATIO	1.166	0.55	1.664	2.34 **
PUBLIC			1.735	1.32
INVENTSHARE			0.421	-1.04
URBAN			1.123	0.3
University Fixed Effets	YES		NO	
Pseudo R-Square	0.178		0.16	
Observations	1215		1215	

Table 3. Government Funding

	Part A			Part B			Part C			Part D		
	Coeff	t-Statistic		Coeff	t-Statistic		Coeff	t-Statistic		Coeff	t-Statistic	
INDFND	0.0258	2.11	**	0.0187	1.57		0.0327	2.47	**	0.0267	2.05	**
LAG_GOVFND	0.1128	7.03	***	0.1105	6.93	***						
LAG_PUB	0.1372	1.51		0.1671	1.85	*	0.1532	1.68	*	0.1807	1.99	**
LAG_PUB_CITES	0.0359	0.94		0.0250	0.65		0.0558	1.42		0.0454	1.15	
TENURE	-0.0885	-0.61		-0.0962	-0.68		-0.0623	-0.39		-0.0752	-0.50	
SPILL	-0.0140	-0.20		-0.0559	-0.76		0.0026	0.03		-0.0292	-0.39	
ENG	0.0798	0.38		0.1054	0.51		0.0934	0.40		0.1048	0.46	
PHYSIC	0.5532	2.37	**	0.5954	2.70	***	0.6659	2.57	*	0.6964	2.82	***
AGE	0.1091	2.06	**	0.1002	1.88	*	0.1541	2.72	***	0.1480	2.62	***
AGESQ	-0.0011	-2.16	**	-0.0010	-1.99	**	-0.0015	-2.75	***	-0.0014	-2.65	***
MALE	0.3511	1.38		0.3442	1.32		0.4531	1.61		0.4545	1.59	
PERIOD	0.0082	0.31		-0.0016	-0.06		0.0246	0.89		0.0224	0.91	
DEPT_QUAL	0.3664	1.94	*	0.2405	1.47		0.3798	1.93	*	0.2724	1.60	
UNIV_RD_EXPEND	-0.3300	-0.50		0.7316	2.55	**	0.0893	0.13		0.6087	2.13	**
AVER_CO_AUTHORS	0.0388	0.58		0.0442	0.66		0.0319	0.49		0.0410	0.64	
STUDENT_FAC_RATIO	-0.1222	-2.57	*	-0.1411	-3.28	***	-0.1271	-2.39	**	-0.1386	-2.86	***
TOT_GRANTS_RATIO	0.0203	0.48		0.0318	0.76		0.0181	0.41		0.0266	0.61	
TOT_FUNDS_RATIO	0.6085	3.69	***	0.6601	4.58	***	0.6954	3.84	***	0.7788	4.86	***
PUBLIC				0.2874	1.51					-0.2690	-1.44	
INVENTSHARE				-0.0171	-1.33					-0.0213	-1.73	*
URBAN				0.5138	3.03	***				0.4708	2.69	***
University Fixed Effets	YES			NO			YES			NO		
Observations	956			956			956			956		

Table 4. Industry Funding

	Part A		Part B		Part C		Part D	
	Coeff	t-Statistic	Coeff	t-Statistic	Coeff	t-Statistic	Coeff	t-Statistic
GOVFND	0.0460	1.32	0.0508	1.43	0.0577	1.76 *	0.0578	1.72 *
LAG_INDFND	0.0906	5.87 ***	0.0901	5.81 ***				
LAG_PUB	0.1853	1.22	0.2223	1.50	0.3303	2.53 ***	0.3523	2.67 ***
LAG_PUB_CITES	-0.0853	-1.26	-0.0965	-1.43	-0.1156	-1.91 *	-0.1163	-1.86 *
TENURE	-0.6663	-3.15 ***	-0.6515	-3.04 ***	-0.6555	-3.06 ***	-0.6269	-2.89 ***
SPILL	-0.6392	-4.22 ***	-0.7004	-4.54 ***	-0.6926	-4.18 ***	-0.7262	-4.12 ***
ENG	-0.2475	-0.70	-0.2104	-0.59	-0.4377	-1.28	-0.3570	-1.01
PHYSIC	-0.4814	-1.48	-0.3785	-1.25	-0.7810	-2.21 ***	-0.6620	-1.94 *
AGE	0.1996	2.27 **	0.1616	1.90 *	0.1876	2.11 ***	0.1555	1.81 *
AGESQ	-0.0019	-2.27 **	-0.0016	-1.91 *	-0.0018	-2.11 ***	-0.0015	-1.82 *
MALE	0.1542	0.31	0.1959	0.39	0.2875	0.58	0.2589	0.53
PERIOD	0.0682	1.48	0.0853	2.16 **	0.0717	1.57	0.0889	2.26 **
DEPT_QUAL	-0.0074	-0.03	-0.3723	-1.95 **	0.0839	0.35	-0.1205	-0.67
UNIV_RD_EXPEND	1.2959	1.13	0.2246	0.46	1.6012	1.39	0.1397	0.31
AVER_CO_AUTHORS	0.1475	2.15 ***	0.1746	2.56 ***	0.1155	1.61	0.1271	1.79 *
STUDENT_FAC_RATIO	0.1065	1.76 *	0.0590	1.15	0.1386	2.14 ***	0.0869	1.59
TOT_GRANTS_RATIO	-0.2037	-2.55 ***	-0.1772	-2.21 **	-0.2249	-2.70 ***	-0.2093	-2.58 ***
TOT_FUNDS_RATIO	0.5091	2.75 ***	0.3031	1.84 *	0.5051	2.65 ***	0.1723	0.95
PUBLIC			0.4143	1.41			0.2691	0.97
INVENTSHARE			-0.0208	-1.50			-0.0125	-1.01
URBAN			0.4574	1.80 *			0.5202	2.01 **
University Fixed Effets	YES		NO		YES		NO	
Observations	956		956		956		956	