Homework set # 3

Due on 2/6

0. The following problems from Artin "Algebra" edition 2: 15.3.9, 15.3.10

1. (a) Let K be an extension of F of degree n. For any $\alpha \in K$ prove that α acting by left multiplication on K is an F-linear transformation of K.

(b) Show that if the matrix of the linear transformation "multipication by α" is the matrix A then α is a root of the characteristic polynomial for A.

(c) Use the procedure in part (b) to find the monic polynomials of degree 3 satisfied by $\sqrt[3]{2}$ and by $1 + \sqrt[3]{2} + \sqrt[3]{4}$.

(d) The polynomials found in part (c) are in fact the irreducible polynomials for these elements (Why?). Give an example (of a K, F, and an α) where the procedure in part (b) does not produce the irreducible polynomial of α over F.

2. Use the fact that $\alpha = 2\cos(2\pi/7)$ satisfies the equation $x^3 + x^2 - 2x - 1 = 0$ to prove that the regular 7-gon is not constructible by straightedge and compass.

3. Use the fact that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$ to conclude that the regular $5 - gon$ is constructible by straightedge and compass.