```
> restart;
```

Euler

```
> #y'=texp(3t)-2y, 0<=t<=1, y(0)=0,
> f := (t,y)-> t*exp(3*t)-2*y;
```

$$f := (t, y) \to t\, e^{3\,t} - 2\, y \tag{1}$$

```
> g:= t -> t*exp(3*t)/5-exp(3*t)/25 +exp(-2*t)/25;
```

$$g := t \to \frac{1}{5}\, t\, e^{3\,t} - \frac{1}{25}\, e^{3\,t} + \frac{1}{25}\, e^{-2\,t} \tag{2}$$

```
> diff(g(t),t)-f(t,g(t));
```

$$0 \tag{3}$$

```
> N := 100.0;
  h:=1/N:
```

$$N := 100.0 \tag{4}$$

```
> w:=0;
  for j from 0 to N-1 do
  w:= w+ f(j*h,w)*h:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.173246481$$

$$err := 0.045852837 \tag{5}$$

```
> N := 200.0;
  h:=1/N:
```

$$N := 200.0 \tag{6}$$

```
> w:=0;
  for j from 0 to N-1 do
  w:= w+ f(j*h,w)*h:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.196176306$$

$$err := 0.022923012 \tag{7}$$

modified Euler -- a 2nd order runge-kutta method

```
> N := 10.0;
  h:=1/N:
```

$$N := 10.0 \tag{8}$$

```
> w:=0;
  for j from 0 to N-1 do
  w:= w+ (f(j*h,w)+f((j+1)*h,w+h*f(j*h,w))          )*h/2:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.297890507$$

$$err := 0.078791189 \tag{9}$$

```
> N := 20.0;
  h:=1/N:
```

$$N := 20.0 \tag{10}$$

```
> w:=0;
  for j from 0 to N-1 do
```

```
w:= w+ (f(j*h,w)+f((j+1)*h,w+h*f(j*h,w))          )*h/2:
od:
w;
err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.238309128$$

$$err := 0.019209810 \quad (11)$$

```
> N := 40.0;
  h:=1/N:
```

$$N := 40.0 \quad (12)$$

```
> w:=0;
  for j from 0 to N-1 do
  w:= w+ (f(j*h,w)+f((j+1)*h,w+h*f(j*h,w))          )*h/2:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.223835120$$

$$err := 0.004735802 \quad (13)$$

Error in Programming

```
> N := 20.0;
  h:=1/N:
```

$$N := 20.0 \quad (14)$$

```
> w:=0;
  for j from 1 to N do
  w:= w+ (f(j*h,w)+f((j+1)*h,w+h*f(j*h,w))          )*h/2:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.994968783$$

$$err := 0.775869465 \quad (15)$$

```
> N := 40.0;
  h:=1/N:
```

$$N := 40.0 \quad (16)$$

```
> w:=0;
  for j from 1 to N do
  w:= w+ (f(j*h,w)+f((j+1)*h,w+h*f(j*h,w))          )*h/2:
  od:
  w;
  err:=abs(w-g(1.0));
```

$$w := 0$$

$$3.582532397$$

$$err := 0.363433079 \quad (17)$$

Runge-Kutta--order 4

```
> N := 10.0;
  h:=1/N:
```

$$N := 10.0 \quad (18)$$

```
> w:=0;
  for j from 0 to N-1 do
  k1:= h*f(j*h,w):
  k2:= h*f((j+0.5)*h,w+k1/2):
  k3:= h*f((j+0.5)*h,w+k2/2):
  k4:= h*f((j+1)*h,w+k3):
  w:= w+ (k1+2*k2+2*k3+k4)/6:
```

```
od:
w10:=w;
err10:=abs(w10-g(1.0));
```

$$w := 0$$

$$w10 := 3.219283394$$

$$err10 := 0.000184076 \quad \textbf{(19)}$$

```
> N := 20.0;
h:=1/N:
```

$$N := 20.0 \quad \textbf{(20)}$$

```
> w:=0;
for j from 0 to N-1 do
k1:= h*f(j*h,w):
k2:= h*f((j+0.5)*h,w+k1/2):
k3:= h*f((j+0.5)*h,w+k2/2):
k4:= h*f((j+1)*h,w+k3):
w:= w+ (k1+2*k2+2*k3+k4)/6:
od:
w20:=w;
err20:=abs(w20-g(1.0));
```

$$w := 0$$

$$w20 := 3.219110831$$

$$err20 := 0.000011513 \quad \textbf{(21)}$$

```
> err10/err20;#Runge-Kutta trounces the earlier methods
```

$$15.98853470 \quad \textbf{(22)}$$