```
> restart;
> a:= 1;
  b:=2;
  alpha:= sin(1);
  beta:= sin(2);
  p := x-> x^2;
  q:= x-> (x-1)^2;
  g:= x->-2*x*cos(x)+x^2*sin(x)+(x-1)^2*sin(x);
  L:= u->-diff((p(x)*diff(u(x),x)),x)+q(x)*u(x);
  U:= sin;#solution
  simplify(L(U)-g(x));
```

$$a := 1$$

$$b := 2$$

$$\alpha := \sin(1)$$

$$\beta := \sin(2)$$

$$p := x \to x^2$$

$$q := x \to (x-1)^2$$

$$g := x \to -2\,x\cos(x) + x^2\sin(x) + (x-1)^2\sin(x)$$

$$L := u \to -\left(\frac{\mathrm{d}}{\mathrm{d}x}\left(p(x)\left(\frac{\mathrm{d}}{\mathrm{d}x}\,u(x)\right)\right)\right) + q(x)\,u(x)$$

$$U := \sin$$

$$0 \qquad \textbf{(1)}$$

```
> l:= x-> alpha+(x-a)/(b-a)*(beta-alpha);
```

$$l := x \to \alpha + \frac{(x-a)\,(\beta-\alpha)}{b-a} \qquad \textbf{(2)}$$

```
> L(U-l);
  U(1)-l(1);
  U(2)-l(2);
```

$$-2\,x\,(\cos(x) - \sin(2) + \sin(1)) + x^2\sin(x) + (x-1)^2\,(\sin(x) - \sin(1) - (x-1)\,(\sin(2) - \sin(1)))$$

$$0$$

$$0 \qquad \textbf{(3)}$$

```
> lprint(L(U-l));
-2*x*(cos(x)-sin(2)+sin(1))+x^2*sin(x)+(x-1)^2*(sin(x)-sin(1)-
(x-1)*(sin(2)-sin(1)))
```

New RHS of diff eq with 0 boundary values.

```
> G:= x-> -2*x*(cos(x)-sin(2)+sin(1))+x^2*sin(x)+(x-1)^2*(sin(x)-
  sin(1)-(x-1)*(sin(2)-sin(1)));#new RHS of diff eq
```

$$G := x \to -2\,x\,(\cos(x) - \sin(2) + \sin(1)) + x^2\sin(x) + (x-1)^2\,(\sin(x) - \sin(1) - (x-1)\,(\sin(2) - \sin(1))) \qquad \textbf{(4)}$$

```
> simplify(L(U-l)-G(x));
```

$$0 \qquad \textbf{(5)}$$

```
> with(CurveFitting);
```

$$[\mathit{ArrayInterpolation}, \mathit{BSpline}, \mathit{BSplineCurve}, \mathit{Interactive}, \mathit{LeastSquares}, \mathit{PolynomialInterpolation}, \mathit{RationalInterpolation}, \mathit{Spline}, \mathit{ThieleInterpolation}] \qquad \textbf{(6)}$$

```
> N:= 5;
```

```
h:=1.0/(N+1);
j:='j';
> for j from 1 to N do
kn||j := [1+(j-1)*h,1+j*h,1+(j+1)*h];
od;

> for j from 1 to N do
phi||j:= unapply(BSpline(2,x,knots=kn||j),x):
od:
```

$$N := 5$$

$$h := 0.1666666667$$

$$j := j$$

$$kn1 := [1., 1.166666667, 1.333333333]$$

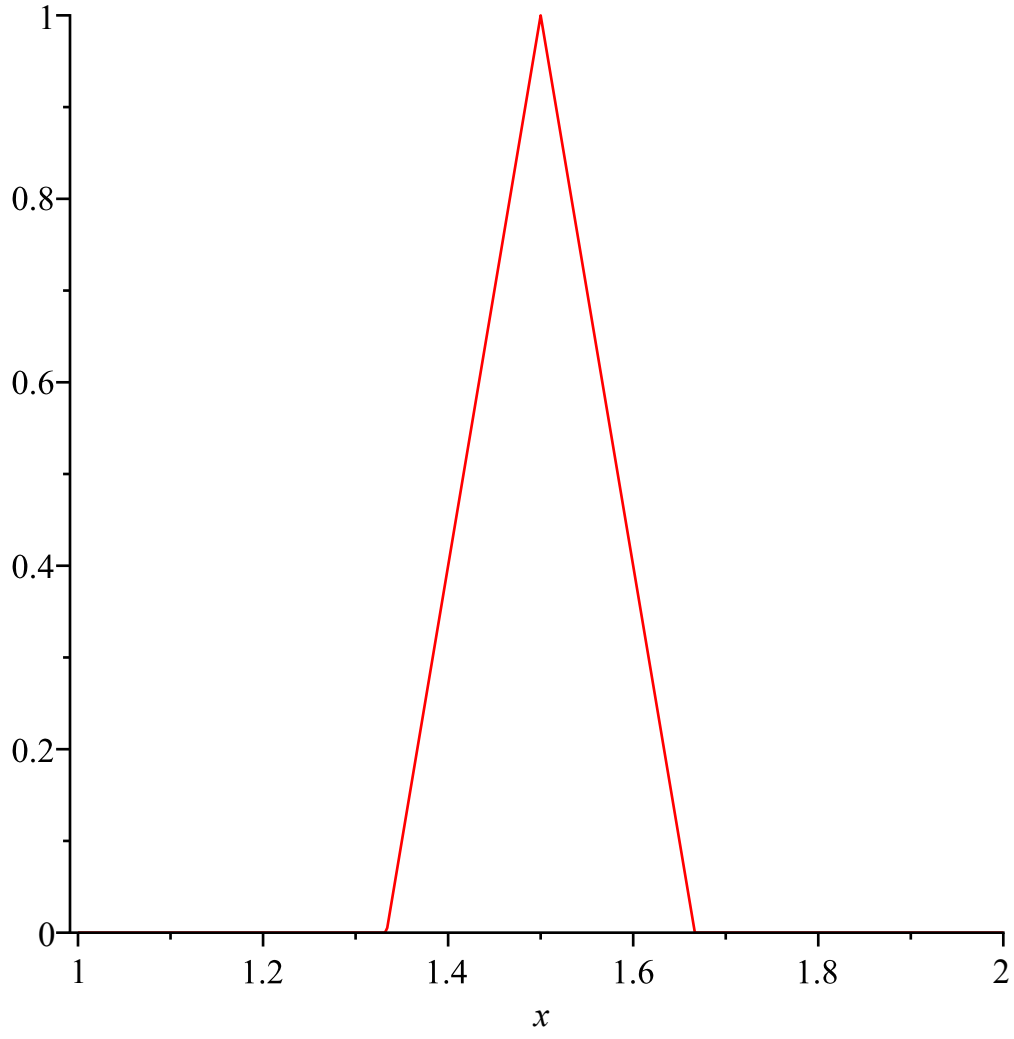
$$kn2 := [1.166666667, 1.333333333, 1.500000000]$$

$$kn3 := [1.333333333, 1.500000000, 1.666666667]$$

$$kn4 := [1.500000000, 1.666666667, 1.833333334]$$

$$kn5 := [1.666666667, 1.833333334, 2.000000000] \quad \textbf{(7)}$$

```
> plot(phi3(x),x=1..2);
```




```
> with(LinearAlgebra):
```

```
> M:= Matrix(N);
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

```
> for j from 1 to N do for i from 1 to N do
  M(i,j):= int(p(x)*diff(phi||j(x),x)*diff(phi||i(x),x),x=a..b)-int
  (q(x)*phi||j(x)*phi||i(x),x=a..b):
  od:
  od:
```

```
> M;
```

$$\begin{bmatrix} 16.44104941 & -9.390663617 & 0. & 0. & 0. \\ -9.390663617 & 21.43179013 & -12.06041664 & 0. & 0. \\ 0. & -12.06041664 & 27.08302465 & -15.06504627 & 0. \\ 0. & 0. & -15.06504627 & 33.39475292 & -18.40455232 \\ 0. & 0. & 0. & -18.40455232 & 40.36697537 \end{bmatrix} \quad (9)$$

```
> N:= 20;
  h:=1.0/(N+1);
> for j from 1 to N do
  kn||j:= [1+(j-1)*h,1+j*h,1+(j+1)*h];
  od;
> for j from 1 to N do
  phi||j:= unapply(BSpline(2,x,knots=kn||j),x):
  od:
  M:= Matrix(N);
  for j from 1 to N do for i from 1 to N do
  if (abs(j-i)<=1.1) then M(i,j):= int(p(x)*diff(phi||j(x),x)*diff
  (phi||i(x),x),x=1+min(i-1,j-1)*h..1+max(i+1,j+1)*h)
  +int(q(x)*phi||j(x)*phi||i(x),x=1+min(i-1,j-1)*h..1+max(i+1,j+1)*
  h): fi:
  od:
  od:
```

$$N := 20$$

$$h := 0.04761904762$$

$$kn1 := [1., 1.047619048, 1.095238095]$$

$$kn2 := [1.047619048, 1.095238095, 1.142857143]$$

$$kn3 := [1.095238095, 1.142857143, 1.190476190]$$

$$kn4 := [1.142857143, 1.190476190, 1.238095238]$$

$$kn5 := [1.190476190, 1.238095238, 1.285714286]$$

$$kn6 := [1.238095238, 1.285714286, 1.333333333]$$

$$kn7 := [1.285714286, 1.333333333, 1.380952381]$$

$$kn8 := [1.333333333, 1.380952381, 1.428571429]$$

$$kn9 := [1.380952381, 1.428571429, 1.476190476]$$
$$kn10 := [1.428571429, 1.476190476, 1.523809524]$$
$$kn11 := [1.476190476, 1.523809524, 1.571428571]$$
$$kn12 := [1.523809524, 1.571428571, 1.619047619]$$
$$kn13 := [1.571428571, 1.619047619, 1.666666667]$$
$$kn14 := [1.619047619, 1.666666667, 1.714285714]$$
$$kn15 := [1.666666667, 1.714285714, 1.761904762]$$
$$kn16 := [1.714285714, 1.761904762, 1.809523810]$$
$$kn17 := [1.761904762, 1.809523810, 1.857142857]$$
$$kn18 := [1.809523810, 1.857142857, 1.904761905]$$
$$kn19 := [1.857142857, 1.904761905, 1.952380952]$$
$$kn20 := [1.904761905, 1.952380952, 2.000000000]$$

$$M := \begin{bmatrix} 20 \text{ x } 20 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (10)$$

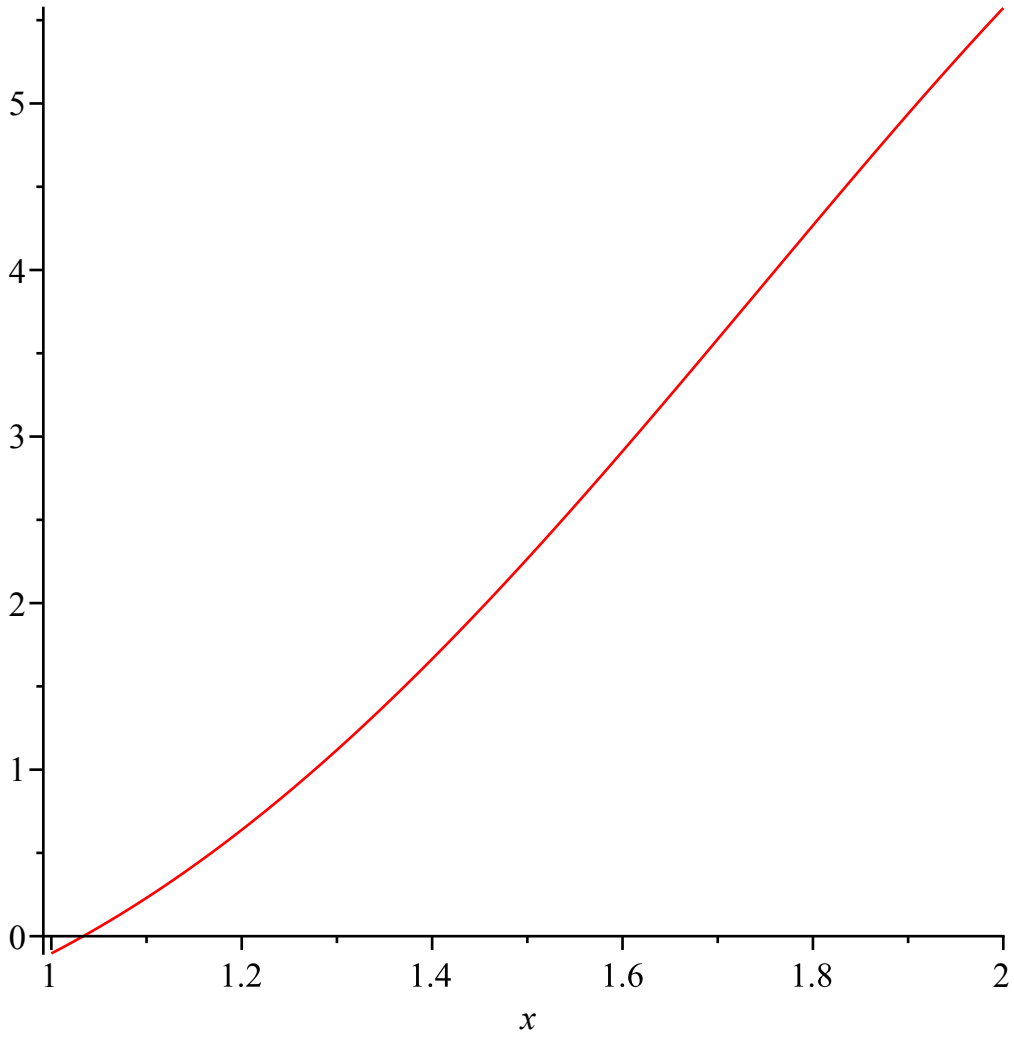
```
> v:=Vector(N):
  for i from 1 to N do
  v(i):= int(G(x)*phi||i(x),x=1+(i-1)*h..1+(i+1)*h):
  od:
```

```
> v;
```

$$\begin{bmatrix} 1 .. 20 \text{ Vector}_{column} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad (11)$$

```
> trueSol:=Vector(N):
  for j from 1 to N do
  trueSol(j):= evalf(U(1+j*h)-l(1+j*h)):
  od:
```

```
> #for j from 1 to N do trueSol(j); od;
  plot(G(x),x=1..2);
```




```
> solution:= LinearSolve(M,v);
```

$$solution := \begin{bmatrix} 1 \,..\, 20\ Vector_{column} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix} \quad \textbf{(12)}$$

```
> Norm(solution-trueSol)/Norm(trueSol);
  Norm(solution-trueSol,1)/Norm(trueSol,1);
```

$$0.0002263772895$$
$$0.0002182122468 \quad \textbf{(13)}$$

```
> N:= 40;
  h:=1.0/(N+1);
> for j from 1 to N do
  kn||j:= [1+(j-1)*h,1+j*h,1+(j+1)*h];
  od:
> for j from 1 to N do
  phi||j:= unapply(BSpline(2,x,knots=kn||j),x):
  od:
  M:= Matrix(N):
```

```
  for j from 1 to N do for i from 1 to N do
  if (abs(j-i)<=1.1) then M(i,j):= int(p(x)*diff(phi||j(x),x)*diff
  (phi||i(x),x),x=1+min(i-1,j-1)*h..1+max(i+1,j+1)*h)
  +int(q(x)*phi||j(x)*phi||i(x),x=1+min(i-1,j-1)*h..1+max(i+1,j+1)*
  h): fi:
  od:
  od:
> v:=Vector(N):
  for i from 1 to N do
  v(i):= int(G(x)*phi||i(x),x=1+(i-1)*h..1+(i+1)*h):
  od:
> trueSol:=Vector(N):
  for j from 1 to N do
  trueSol(j):= evalf(U(1+j*h)-l(1+j*h)):
  od:
> #for j from 1 to N do trueSol(j); od;
  #plot(G(x),x=1..2);
> solution:= LinearSolve(M,v);
> Norm(solution-trueSol)/Norm(trueSol);
  Norm(solution-trueSol,1)/Norm(trueSol,1);
```

$$N := 40$$

$$h := 0.02439024390$$

$$solution := \begin{bmatrix} 1 .. 40\ Vector_{column} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

$$0.00005980785087$$

$$0.00005764963625 \qquad \textbf{(14)}$$