```
> restart;
```

Problem 2.1

```
> f:= x->x^6;
  Digits:=40;
  xx :=sqrt(2.0);
```

$$f := x \to x^6$$

$$Digits := 40$$

$$xx := 1.4142135623730950488016887242096980785 70 \tag{1}$$

```
> N:= h -> (f(xx+h)-f(xx))/h;
```

$$N := h \to \frac{f(xx+h) - f(xx)}{h} \tag{2}$$

```
> N2:= unapply(2*N(h/2)-N(h),h);
```

$$N2 := h \to \frac{1}{h}\left(4\left(\left(1.4142135623730950488016887242096980785 70 + \frac{1}{2}\,h\right)^6 - 8.0000000000000000000000000000000000000011\right)\right) - \frac{1}{h}\left((1.4142135623730950488016887242096980785 70 + h)^6 - 8.0000000000000000000000000000000000000011\right) \tag{3}$$

```
> for j from 1 to 30 do
  print(j,N2(10^(-j))-6*xx^5):
  od:
```

1, -0.30609454959486488466095863142214970718

2, -0.00285100146470821468509343953707741711

3, -0.00002830677867302060343478278069305971

4, $-2.828652132170905050062126454557 1\ 10^{-7}$

5, $-2.828449624820436403377964785 71\ 10^{-9}$

6, $-2.828429374746932559817388571\ 10^{-11}$

7, $-2.828427349746197521938857 1\ 10^{-13}$

8, $-2.828427147246190153885 71\ 10^{-15}$

9, $-2.828427126996175388571\ 10^{-17}$

10, $-2.828427124827538857 1\ 10^{-19}$

11, $-2.828427132753885 71\ 10^{-21}$

12, $-2.828303275388571\ 10^{-23}$

13, $-2.9103275388571\ 10^{-25}$

14, $1.896724611429\ 10^{-26}$

15, $6.1896724611429\ 10^{-25}$

16, $2.061896724611429\ 10^{-23}$

17, $1.7061896724611429\ 10^{-22}$

18, $-1.52938103275388571\ 10^{-21}$

19, $-5.2938103275388571\ 10^{-22}$

20, $5.947061896724611429\ 10^{-20}$

21, $1.7594706189672461142 9\ 10^{-18}$

22, $1.8759470618967246114 29\ 10^{-17}$

$$23,\ 2.875947061896724611429\ 10^{-17}$$

$$24,\ 1.8287594706189672461 1429\ 10^{-15}$$

$$25,\ -1.17124052938103275388571\ 10^{-15}$$

$$26,\ 2.188287594706189672461 1429\ 10^{-13}$$

$$27,\ -2.81171240529381032753 88571\ 10^{-13}$$

$$28,\ 5.718828759470618967246 11429\ 10^{-12}$$

$$29,\ 4.571882875947061896724611429\ 10^{-11}$$

$$30,\ 2.045718828759470618967246 11429\ 10^{-9}$$ **(4)**

Problem 2.2

```
> restart:
> with(LinearAlgebra):
> N:=2;
  v:=Vector(N+1):
> for j from 0 to N do v[j+1]:=j/N; od:
> v;
> L := proc( N::integer, i::integer, v::Vector, x)
>            description "Lagrange function";
> local j,T;
> T:=1:
> for j from 0 to N do
> if (j<>i) then T:= T*(x-v[j+1])/(v[i+1]-v[j+1])
> fi;
> od;
> T;
> end proc;
```

$$N := 2$$

$$\begin{bmatrix} 0 \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

$L := \mathbf{proc}(N::integer, i::integer, v::Vector, x)$ **(5)**

```
    local j, T;
    description "Lagrange function";
    T := 1; for j from 0 to N do
        if j <> i then T := T* (x - v[j + 1]) / (v[i + 1] - v[j + 1]) end if
    end do;
    T
end proc
```

```
> T:=0;
  for j from 0 to 2 do T:= T+ (f||j)*L(2,j,v,x);od;
  p:=  unapply(T,x);
```

$$T := 0$$

$$T := -f0\ (-2\,x + 1)\ (x - 1)$$

$$T := -f0\ (-2\,x + 1)\ (x - 1) - 4\,f1\,x\ (x - 1)$$

$$T := -f0\ (-2\,x + 1)\ (x - 1) - 4\,f1\,x\ (x - 1) + 2\,f2\,x\left(x - \frac{1}{2}\right)$$

$$p := x \to -f0\,(-2x+1)\,(x-1) - 4\,f1\,x\,(x-1) + 2\,f2\,x\left(x-\frac{1}{2}\right) \tag{6}$$

```
> int(p(x),x=0..1);
```

$$\frac{1}{6}f0 + \frac{2}{3}f1 + \frac{1}{6}f2 \tag{7}$$

Problem 2.3

```
> restart:
  with(CurveFitting);
```

[*ArrayInterpolation*, *BSpline*, *BSplineCurve*, *Interactive*, *LeastSquares*, *PolynomialInterpolation*, *RationalInterpolation*, *Spline*, *ThieleInterpolation*] (8)

```
> t:=Vector(3);
  y:=Vector(3);
```

$$t := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

```
> t[1]:= 1;
  t[2]:= 3/2;
  t[3]:= 2;
  for j from 1 to 3 do y[j]:=f||(j-1); od;
```

$$t_1 := 1$$

$$t_2 := \frac{3}{2}$$

$$t_3 := 2$$

$$y_1 := f0$$

$$y_2 := f1$$

$$y_3 := f2 \tag{10}$$

```
> int(PolynomialInterpolation(t,y,x),x=1..2);
```

$$\frac{1}{6}f2 + \frac{2}{3}f1 + \frac{1}{6}f0 \tag{11}$$

Problem 2.4

```
> restart:
  with(CurveFitting);
```

[*ArrayInterpolation*, *BSpline*, *BSplineCurve*, *Interactive*, *LeastSquares*, *PolynomialInterpolation*, *RationalInterpolation*, *Spline*, *ThieleInterpolation*] (12)

```
> t:=Vector(4);
  y:=Vector(4);
  g0:= h-> f0+h*df0;
  g1:= h-> f1+h*df1;
```

$$t := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$g0 := h \rightarrow f0 + h\, df0$$

$$g1 := h \rightarrow f1 + h\, df1 \quad \textbf{(13)}$$

```
> t[1]:= X0+h1;
  t[2]:= X0+h2;
  t[3]:= X1+h3;
  t[4]:= X1+h4;
```

$$t_1 := X0 + h1$$

$$t_2 := X0 + h2$$

$$t_3 := X1 + h3$$

$$t_4 := X1 + h4 \quad \textbf{(14)}$$

```
> for j from 1 to 2 do y[j]:= g0(h||j); od;
  for j from 3 to 4 do y[j]:= g1(h||j); od;
```

$$y_1 := f0 + h1\, df0$$

$$y_2 := f0 + h2\, df0$$

$$y_3 := f1 + h3\, df1$$

$$y_4 := f1 + h4\, df1 \quad \textbf{(15)}$$

```
> limit(limit(limit(limit(PolynomialInterpolation(t,y,x),h4=0),h3=0),h2=0),
  h1=0);
```

$$-\frac{1}{(-X1+X0)^3}\left(X0^2\,X1^2\,df0 - X0^2\,X1^2\,df1 + 3\,X0^2\,X1\,f1 - 3\,X0\,X1^2\,f0 + X1^3\,f0 + 2\,f0\,x^3 - 2\,f1\,x^3 - X0^3\,f1\right. \quad \textbf{(16)}$$
$$- 2\,X1^2\,df0\,x^2 + X1\,df0\,x^3 + X1\,df1\,x^3 - 3\,X1\,f0\,x^2 + 3\,X1\,f1\,x^2 - X1^2\,df1\,x^2 + X1^3\,df0\,x + X0^3\,X1\,df1$$
$$- X0^3\,df1\,x + 2\,X0^2\,df1\,x^2 + X0^2\,df0\,x^2 - 3\,X0\,f0\,x^2 + 3\,X0\,f1\,x^2 - X0\,df0\,x^3 - X0\,df1\,x^3 - X0\,df0\,X1^3$$
$$- 2\,X0^2\,X1\,df0\,x - X0^2\,X1\,df1\,x - 6\,X0\,X1\,f1\,x + 6\,X0\,X1\,f0\,x - X0\,X1\,df1\,x^2 + 2\,X0\,X1^2\,df1\,x$$
$$\left. + X0\,X1^2\,df0\,x + X0\,X1\,df0\,x^2\right)$$

```
> P:= unapply(%,f0,df0,f1,df1,x);
```

$$P := (f0, df0, f1, df1, x) \rightarrow -\frac{1}{(-X1+X0)^3}\left(X0^2\,X1^2\,df0 + X1^3\,f0 + 2\,f0\,x^3 - 2\,f1\,x^3 - X0^3\,f1 - X1^2\,df1\,x^2\right. \quad \textbf{(17)}$$
$$- X0^3\,df1\,x + 3\,X0^2\,X1\,f1 - 3\,X0\,f0\,x^2 + X1^3\,df0\,x - X0\,df0\,x^3 + 2\,X0^2\,df1\,x^2 + 3\,X1\,f1\,x^2 + 3\,X0\,f1\,x^2$$
$$- 3\,X1\,f0\,x^2 - X0^2\,X1^2\,df1 - X0\,df0\,X1^3 + X0^3\,X1\,df1 - 3\,X0\,X1^2\,f0 + X1\,df1\,x^3 - 2\,X1^2\,df0\,x^2$$
$$+ X1\,df0\,x^3 - X0\,df1\,x^3 + X0^2\,df0\,x^2 - 2\,X0^2\,X1\,df0\,x - X0^2\,X1\,df1\,x - 6\,X0\,X1\,f1\,x + 6\,X0\,X1\,f0\,x$$
$$\left. - X0\,X1\,df1\,x^2 + 2\,X0\,X1^2\,df1\,x + X0\,X1^2\,df0\,x + X0\,X1\,df0\,x^2\right)$$

```
> X0:=0;X1:=1;
```

$$X0 := 0$$

$$X1 := 1 \quad \textbf{(18)}$$

```
> P(f0,df0,f1,df1,x);
```

$$f0 - 2\,df0\,x^2 - df1\,x^2 + df0\,x^3 + df1\,x^3 - 3\,f0\,x^2 + 3\,f1\,x^2 + 2\,f0\,x^3 - 2\,f1\,x^3 + df0\,x \tag{19}$$

```
> int(P(f0,df0,f1,df1,x),x=0..1);
```

$$\frac{1}{12}\,df0 - \frac{1}{12}\,df1 + \frac{1}{2}\,f0 + \frac{1}{2}\,f1 \tag{20}$$

```
> P(f0,df0,f1,f1,x);
```

$$f0 - 2\,df0\,x^2 + df0\,x^3 - 3\,f0\,x^2 + 2\,f1\,x^2 + 2\,f0\,x^3 - f1\,x^3 + df0\,x \tag{21}$$

```
> int(P(f0,df0,f1,f1,x),x=0..1);
```

$$\frac{1}{12}\,df0 + \frac{1}{2}\,f0 + \frac{5}{12}\,f1 \tag{22}$$