

```

> Digits:=10;
> u:=evalf(sqrt(2));
f := h -> ((u+h)^4-(u-h)^4)/(2*h)-4*u^3;

> for j from 1 to 10 do
> print(j,f(10^(-j)));
> od;

```

$$u := 1.414213562$$

$$f := h \rightarrow \frac{1}{2} \frac{(u+h)^4 - (u-h)^4}{h} - 4 u^3$$

1, 0.05656855  
 2, 0.00056571  
 3, 0.00000551  
 4, 0.00000151  
 5, -0.00000849  
 6, -0.00020849  
 7, 0.00129151  
 8, -0.01370849  
 9, 0.18629151  
 10, -11.31370849

(1)

```

> Digits:=20;
                                         Digits := 20
(2)

> u:=evalf(sqrt(2));
  f := h -> ((u+h)^4-(u-h)^4)/(2*h)-4*
> u^3;
> for j from 1 to 20 do
> print(j,f(10^(-j)));
> od;

```

$u := 1.4142135623730950488$

$$f := h \rightarrow \frac{1}{2} \frac{(u+h)^4 - (u-h)^4}{h} - 4 u^3$$

1, 0.056568542494923802  
 2, 0.000565685424949240  
 3, 0.000005656854249510  
 4, 5.6568542610  $10^{-8}$   
 5, 5.65684610  $10^{-10}$   
 6, 5.639610  $10^{-12}$   
 7, 2.39610  $10^{-13}$   
 8, 2.39610  $10^{-13}$   
 9, 1.5239610  $10^{-11}$   
 10, 1.5239610  $10^{-11}$   
 11, 1.015239610  $10^{-9}$   
 12, 1.015239610  $10^{-9}$   
 13, 1.015239610  $10^{-9}$   
 14, 0.000001501015239610  
 15, -0.000008498984760390  
 16, 0.000291501015239610  
 17, 0.001291501015239610  
 18, -0.013708498984760390  
 19, 0.186291501015239610  
 20, -11.313708498984760390

(3)

```

> restart;
> f:= x->x^6:
> df := (x,h) -> (f(x+h)-f(x-h))/(2*h):
> RE := (x,h)-> (4*df(x,h/2)-df(x,h))/3.0;

$$RE := (x, h) \rightarrow \frac{4 df\left(x, \frac{1}{2} h\right) - df(x, h)}{3.0} \quad (4)$$


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```

> u:=sqrt(2.0);

u := 1.414213562373095048801688724209698078570 \quad (5)

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```

> Digits:=40;
> for j from 1 to 25 do
> print(j,RE(u,10^(-j))-6*u^5,df(u,10^(-j)/2)-6*u^5);
> od;

```

*Digits := 40*

```

1, -0.00021213203435596425732025330863145471, 0.14147438924589849594449893574812767154
2, -2.121320343559642573202533086311  $10^{-8}$ , 0.00141421886567395394790812173054241389
3, -2.12132034355964257320253304  $10^{-12}$ , 0.00001414213615406103637792753054273029
4, -2.1213203435596425731571  $10^{-16}$ , 1.4142135629034251346915993675429  $10^{-7}$ 
5, -2.121320343559645238  $10^{-20}$ , 1.41421356237839834966058781429  $10^{-9}$ 
6, -2.12132034321904  $10^{-24}$ , 1.414213562373148081810311429  $10^{-11}$ 
7, -2.1212721904  $10^{-28}$ , 1.4142135623730955791611429  $10^{-13}$ 
8, -2.055238  $10^{-32}$ , 1.41421356237309504611429  $10^{-15}$ 
9, 4.1278096  $10^{-31}$ , 1.414213562373124611429  $10^{-17}$ 
10, 5.7944762  $10^{-31}$ , 1.4142135623724611429  $10^{-19}$ 
11, 5.7944762  $10^{-31}$ , 1.41421356724611429  $10^{-21}$ 
12, -1.9942055238  $10^{-28}$ , 1.414196724611429  $10^{-23}$ 
13, -4.36608721904  $10^{-27}$ , 1.3896724611429  $10^{-25}$ 
14, 1.896724611429  $10^{-26}$ , 1.896724611429  $10^{-26}$ 
15, 7.8563391278096  $10^{-25}$ , 6.1896724611429  $10^{-25}$ 
16, 6.1896724611429  $10^{-25}$ , 6.1896724611429  $10^{-25}$ 
17, 1.0395230057944762  $10^{-22}$ , 7.061896724611429  $10^{-23}$ 
18, -6.9604769942055238  $10^{-22}$ , -5.2938103275388571  $10^{-22}$ 
19, -5.2938103275388571  $10^{-22}$ , -5.2938103275388571  $10^{-22}$ 
20, 7.613728563391278096  $10^{-20}$ , 5.947061896724611429  $10^{-20}$ 
21, -2.4052938103275388571  $10^{-19}$ , -2.4052938103275388571  $10^{-19}$ 
22, -1.24052938103275388571  $10^{-18}$ , -1.24052938103275388571  $10^{-18}$ 
23, 2.875947061896724611429  $10^{-17}$ , 2.875947061896724611429  $10^{-17}$ 
24, -1.7124052938103275388571  $10^{-16}$ , -1.7124052938103275388571  $10^{-16}$ 
25, -1.17124052938103275388571  $10^{-15}$ , -1.17124052938103275388571  $10^{-15}$  \quad (6)

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