```
> restart;
> with(student);
```

[D, *Diff*, *Doubleint*, *Int*, *Limit*, *Lineint*, *Product*, *Sum*, *Tripleint*, *changevar*, *completesquare*, *distance*, *equate*, *integrand*, *intercept*, *intparts*, *leftbox*, *leftsum*, *makeproc*, *middlebox*, *middlesum*, *midpoint*, *powsubs*, *rightbox*, *rightsum*, *showtangent*, *simpson*, *slope*, *summand*, *trapezoid*] **(1)**

```
> f := x -> sin(x);
```

$$f := x \to \sin(x) \quad \textbf{(2)}$$

```
> for j from 1 to 5 do
  evalf(trapezoid(f(x),x=0..1,2^j))+evalf(cos(1))-1.0;
  od;
```

$$-0.0096171786$$
$$-0.0023967565$$
$$-0.0005987207$$
$$-0.0001496509$$
$$-0.0000374109 \quad \textbf{(3)}$$

```
> for j from 1 to 5 do
  evalf(trapezoid(f(x)^8,x=0..1,2^j)-int(f(x)^8,x=0..1));
  od;
```

$$0.0272201901$$
$$0.0067560001$$
$$0.0016833022$$
$$0.0004204474$$
$$0.0001050879 \quad \textbf{(4)}$$

An aside: why is this so very good an approximation?

```
> for j from 1 to 5 do
  evalf(trapezoid(f(x),x=0..2*Pi,2^j))+evalf(cos(2*Pi))-1.0;
  od;
```

$$0.$$
$$0.$$
$$0.$$
$$0.$$
$$0. \quad \textbf{(5)}$$

```
> Digits:=40;
  for j from 1 to 5 do
  evalf(trapezoid(f(x)^4,x=0..2*Pi,2^j)-int(f(x)^4,x=0..2*Pi));
  od;
```

$$Digits := 40$$
$$-2.356194490192344928846982537459627163148$$
$$0.785398163397448309615660845819875721049$$
$$0.$$
$$0.$$
$$0. \quad \textbf{(6)}$$

```
> for j from 1 to 5 do
  j;abs(evalf(trapezoid(f(x),x=0..1,2^(j+1))-int(f(x),x=0..1)))/
  (abs(evalf(trapezoid(f(x),x=0..1,2^j)-trapezoid(f(x),x=0..1,2^(j+1))))/3);
  od;
```

$$1$$
$$0.99582401332355597342931546956376898015 20$$
$$2$$
$$0.99895775174322780803951061732933378603 57$$

$$3$$

$$0.99973954699815207380161769023712615688904$$

$$4$$

$$0.99993489356260628226414567773418737674085$$

$$5$$

$$0.99998372381641635751673875489574029712017 \quad (7)$$

```
> Digits:=20;
  f:= x -> x^(1/2);
```

$$Digits := 20$$

$$f := x \to \sqrt{x} \quad (8)$$

```
> for j from 1 to 5 do
  j;print(abs(evalf(trapezoid(f(x),x=0..1,2^(j+1))-int(f(x),x=0..1))),
  (abs(evalf(trapezoid(f(x),x=0..1,2^j)-trapezoid(f(x),x=0..1,2^(j+1))))/3));
  od;
```

$$1$$

$$0.023383620423920123870, 0.013243218549824260200$$

$$2$$

$$0.0085364450422123370 1, 0.0049490584605692622867$$

$$3$$

$$0.0030854697894385079 8, 0.0018169917509246096767$$

$$4$$

$$0.0011077303877248972 3, 0.0006592464672378702500 0$$

$$5$$

$$0.0003958552881597360 5, 0.0002372916998550537266 7 \quad (9)$$

```
> Digits:=20;
  f:= x -> x^(7/2);
```

$$Digits := 20$$

$$f := x \to x^{7/2} \quad (10)$$

```
> for j from 1 to 5 do
  j;print(abs(evalf(trapezoid(f(x),x=0..1,2^(j+1))-int(f(x),x=0..1))),
  (abs(evalf(trapezoid(f(x),x=0..1,2^j)-trapezoid(f(x),x=0..1,2^(j+1))))/3));
  od;
```

$$1$$

$$0.018166606495247401438, 0.017935115035563198873$$

$$2$$

$$0.0045532241068512258 0, 0.0045377941294653918800$$

$$3$$

$$0.0011390616963063598 4, 0.0011380541368482886533$$

$$4$$

$$0.0002848140930760894 3, 0.0002847492010767568033 3$$

$$5$$

$$0.0000712066288351968 9, 0.0000712024880802975133 33 \quad (11)$$

```
> Digits:=20;
  f:= x -> x^(3/2);
```

$$Digits := 20$$

$$f := x \to x^{3/2} \quad (12)$$

```
> for j from 1 to 5 do
  j;print(abs(evalf(trapezoid(f(x),x=0..1,2^(j+1))-int(f(x),x=0..1))),
  (abs(evalf(trapezoid(f(x),x=0..1,2^j)-trapezoid(f(x),x=0..1,2^(j+1))))/3));
  od;
```

$$\begin{matrix} 1 \\ 0.0070181108579006868 2, 0.0065861948129120647600 \\ 2 \\ 0.0018124647999742328 8, 0.0017352153526421513133 \\ 3 \\ 0.0004634013020478849 2, 0.0004496878326421159 8667 \\ 4 \\ 0.0001176712097782403 8, 0.0001152433640898815 1333 \\ 5 \\ 0.0000297398625287048 0, 0.0000293104490831785 26667 \end{matrix} \tag{13}$$