```
> restart;
```

Homework 10

```
> with(LinearAlgebra):
```

Problem 1ad, 3ad on pg 599

```
> A:= Matrix([ [2.0,1.0,1.0],
               [1.0,2.0,1.0],
               [1.0,1.0,2.0] ]);
```

$$A := \begin{bmatrix} 2.0 & 1.0 & 1.0 \\ 1.0 & 2.0 & 1.0 \\ 1.0 & 1.0 & 2.0 \end{bmatrix} \quad (1)$$

```
> x:=Vector([-1.0,2.0,1.0]);
```

$$x := \begin{bmatrix} -1.0 \\ 2.0 \\ 1.0 \end{bmatrix} \quad (2)$$

```
> for j from 1 to 5 do
  y:= x/norm(x,infinity);
  x := (A . y);
  print(y,x);
  od:
```

$$\begin{bmatrix} -0.500000000000000 \\ 1. \\ 0.500000000000000 \end{bmatrix}, \begin{bmatrix} 0.500000000000000 \\ 2. \\ 1.50000000000000 \end{bmatrix}$$

$$\begin{bmatrix} 0.250000000000000 \\ 1. \\ 0.750000000000000 \end{bmatrix}, \begin{bmatrix} 2.25000000000000 \\ 3. \\ 2.75000000000000 \end{bmatrix}$$

$$\begin{bmatrix} 0.749999999925000 \\ 0.999999999900000 \\ 0.916666666575000 \end{bmatrix}, \begin{bmatrix} 3.41666666632500 \\ 3.66666666630000 \\ 3.58333333297500 \end{bmatrix}$$

$$\begin{bmatrix} 0.931818181973485 \\ 1.00000000016667 \\ 0.977272727435606 \end{bmatrix}, \begin{bmatrix} 3.84090909154924 \\ 3.90909090974243 \\ 3.88636363701136 \end{bmatrix}$$

$$\begin{bmatrix} 0.982558139359215 \\ 0.999999999821212 \\ 0.994186046333880 \end{bmatrix}, \begin{bmatrix} 3.95930232487352 \\ 3.97674418533552 \\ 3.97093023184819 \end{bmatrix} \quad (3)$$

As a comparison

```
> Eigenvalues(A);
```

$$\begin{bmatrix} 1. + 0.\,\mathrm{I} \\ 4.00000000000000 + 0.\,\mathrm{I} \\ 1.00000000000000 + 0.\,\mathrm{I} \end{bmatrix} \quad (4)$$

```
> A:= Matrix([ [4.0,1.0,1.0,1.0],
               [1.0,3.0,-1.0,1.0],
               [1.0,-1.0,2.0,0.0],
               [1.0,1.0,0.0,2.0] ]);
```

$$A := \begin{bmatrix} 4.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 3.0 & -1.0 & 1.0 \\ 1.0 & -1.0 & 2.0 & 0. \\ 1.0 & 1.0 & 0. & 2.0 \end{bmatrix} \quad (5)$$

```
> x:=Vector([1.0,-2.0,0.0,2.0]);
```

$$x := \begin{bmatrix} 1.0 \\ -2.0 \\ 0. \\ 2.0 \end{bmatrix} \quad (6)$$

```
> for j from 1 to 10 do
  y:= x/norm(x,infinity);
  x := (A . y);
  print(y,x);
  od:
```

$$\begin{bmatrix} 1.00000000013169 \\ 0.268857010678627 \\ 0.353077279083981 \\ 0.366959740909039 \end{bmatrix}, \begin{bmatrix} 4.98889403119842 \\ 1.82045349399263 \\ 1.43729754762103 \\ 2.00277649262840 \end{bmatrix}$$

$$\begin{bmatrix} 1.00000000005474 \\ 0.364901215120614 \\ 0.288099434205548 \\ 0.401446990097107 \end{bmatrix}, \begin{bmatrix} 5.05444763964222 \\ 2.20805120130814 \\ 1.21129765334522 \\ 2.16779519536957 \end{bmatrix}$$

$$\begin{bmatrix} 0.999999999999790 \\ 0.436853116053641 \\ 0.239649856859672 \\ 0.428888644204564 \end{bmatrix}, \begin{bmatrix} 5.10539161711704 \\ 2.49979813550560 \\ 1.04244659766549 \\ 2.29463040446256 \end{bmatrix}$$

$$\left[\begin{matrix}0.999999999887852\\0.489638860776927\\0.204185432916358\\0.449452378248894\end{matrix}\right], \left[\begin{matrix}5.14327667149359\\2.71418352755117\\0.918732004943641\\2.38854361716257\end{matrix}\right]$$

$$\left[\begin{matrix}0.999999999844404\\0.527714859706480\\0.178627762704800\\0.464401153068302\end{matrix}\right], \left[\begin{matrix}5.17074377485720\\2.86891796932735\\0.829540665547523\\2.45651716568749\end{matrix}\right]$$

$$\left[\begin{matrix}1.00000000018201\\0.554836614376380\\0.160429659990533\\0.475080041304587\end{matrix}\right], \left[\begin{matrix}5.19034631639953\\2.97916022462520\\0.766022705786694\\2.50499669716756\end{matrix}\right]$$

$$\left[\begin{matrix}1.00000000022470\\0.573981010839606\\0.147586049034624\\0.482626118765062\end{matrix}\right], \left[\begin{matrix}5.20419317953810\\3.05698310247396\\0.721191087454344\\2.53923324859443\end{matrix}\right]$$

$$\left[\begin{matrix}0.999999999754331\\0.587407691502004\\0.138578846402696\\0.487920636373454\end{matrix}\right], \left[\begin{matrix}5.21390717329548\\3.11156486423110\\0.689750001057719\\2.56324896400324\end{matrix}\right]$$

$$\left[\begin{matrix}0.999999999884221\\0.596781791553103\\0.132290425980465\\0.491617682960466\end{matrix}\right], \left[\begin{matrix}5.22068990003092\\3.14967263152353\\0.667799060292049\\2.58001715735826\end{matrix}\right]$$

$$\left[\begin{matrix}1.00000000024570\\0.603305825974982\\0.127913948777569\\0.494190845921893\end{matrix}\right], \left[\begin{matrix}5.22541062165724\\3.17619437531497\\0.652522071825854\\2.59168751806447\end{matrix}\right] \quad \textbf{(7)}$$

As a comparison

```
> Eigenvalues(A);
```

$$\left[\begin{matrix}5.91963956583942 + 0.\,I\\2.54018021708029 + 0.688172819843839\,I\\2.54018021708029 - 0.688172819843839\,I\end{matrix}\right] \quad \textbf{(8)}$$

```
> A:= Matrix([ [4.0,2.0,1.0],
               [0.0,3.0,2.0],
               [1.0,1.0,4.0] ]);
```

$$A := \begin{bmatrix} 4.0 & 2.0 & 1.0 \\ 0. & 3.0 & 2.0 \\ 1.0 & 1.0 & 4.0 \end{bmatrix} \quad (9)$$

```
> x:=Vector([1.0,2.0,1.0]);
```

$$x := \begin{bmatrix} 1.0 \\ 2.0 \\ 1.0 \end{bmatrix} \quad (10)$$

```
> for j from 1 to 7 do
  y:= x/norm(x,infinity);
  x := (A . y);
  print(y,x);
  od:
```

$$\begin{bmatrix} 0.500000000000000 \\ 1. \\ 0.500000000000000 \end{bmatrix}, \begin{bmatrix} 4.50000000000000 \\ 4. \\ 3.50000000000000 \end{bmatrix}$$

$$\begin{bmatrix} 0.999999999900000 \\ 0.888888888800000 \\ 0.777777777700000 \end{bmatrix}, \begin{bmatrix} 6.55555555490000 \\ 4.22222222180000 \\ 4.99999999950000 \end{bmatrix}$$

$$\begin{bmatrix} 1.00000000002222 \\ 0.644067796624482 \\ 0.762711864423729 \end{bmatrix}, \begin{bmatrix} 6.05084745776158 \\ 3.45762711872090 \\ 4.69491525434162 \end{bmatrix}$$

$$\begin{bmatrix} 0.999999999764595 \\ 0.571428571294054 \\ 0.775910363963005 \end{bmatrix}, \begin{bmatrix} 5.91876750560949 \\ 3.26610644180817 \\ 4.67507002691067 \end{bmatrix}$$

$$\begin{bmatrix} 0.999999999731262 \\ 0.551822053803432 \\ 0.789872219380727 \end{bmatrix}, \begin{bmatrix} 5.89351632591264 \\ 3.23521060017175 \\ 4.71131093105760 \end{bmatrix}$$

$$\begin{bmatrix} 1.00000000008758 \\ 0.548944029599189 \\ 0.799405765748965 \end{bmatrix}, \begin{bmatrix} 5.89729382529766 \\ 3.24564362029550 \\ 4.74656709268263 \end{bmatrix}$$

(11)

$$\begin{bmatrix} 1.00000000000355 \\ 0.550361524532518 \\ 0.804872070700989 \end{bmatrix}, \begin{bmatrix} 5.90559511978023 \\ 3.26082871499953 \\ 4.76984980734003 \end{bmatrix} \quad (11)$$

As a comparison

```
> Eigenvalues(A);
```

$$\begin{bmatrix} 5.91963956583942 + 0.\,I \\ 2.54018021708029 + 0.688172819843839\,I \\ 2.54018021708029 - 0.688172819843839\,I \end{bmatrix} \quad (12)$$

```
> A:= Matrix([ [-4.0,0.0,0.5,0.5],
               [0.5,-2.0,0.0,0.5],
               [0.5,0.5,0.0,0.0],
               [0.0,1.0,1.0,4.0] ]);
```

$$A := \begin{bmatrix} -4.0 & 0. & 0.5 & 0.5 \\ 0.5 & -2.0 & 0. & 0.5 \\ 0.5 & 0.5 & 0. & 0. \\ 0. & 1.0 & 1.0 & 4.0 \end{bmatrix} \quad (13)$$

```
> x:=Vector([0.0,0.0,0.0,1.0]);
```

$$x := \begin{bmatrix} 0. \\ 0. \\ 0. \\ 1.0 \end{bmatrix} \quad (14)$$

```
> Digits:=10;
  for j from 1 to 3 do
  y:= x/norm(x,infinity);
  x := (A . y);
  print(y,x);
  od:
```

$$Digits := 10$$

$$\begin{bmatrix} 0. \\ 0. \\ 0. \\ 1. \end{bmatrix}, \begin{bmatrix} 0.500000000000000 \\ 0.500000000000000 \\ 0. \\ 4. \end{bmatrix}$$

$$\begin{bmatrix} 0.125000000000000 \\ 0.125000000000000 \\ 0. \\ 1. \end{bmatrix}, \begin{bmatrix} 0. \\ 0.312500000000000 \\ 0.125000000000000 \\ 4.12500000000000 \end{bmatrix}$$

$$\left[\begin{matrix} 0. \\ 0.0757575757500000 \\ 0.0303030303000000 \\ 0.999999999900000 \end{matrix}\right], \left[\begin{matrix} 0.515151515100000 \\ 0.348484848450000 \\ 0.0378787878750000 \\ 4.10606060565000 \end{matrix}\right] \quad \textbf{(15)}$$

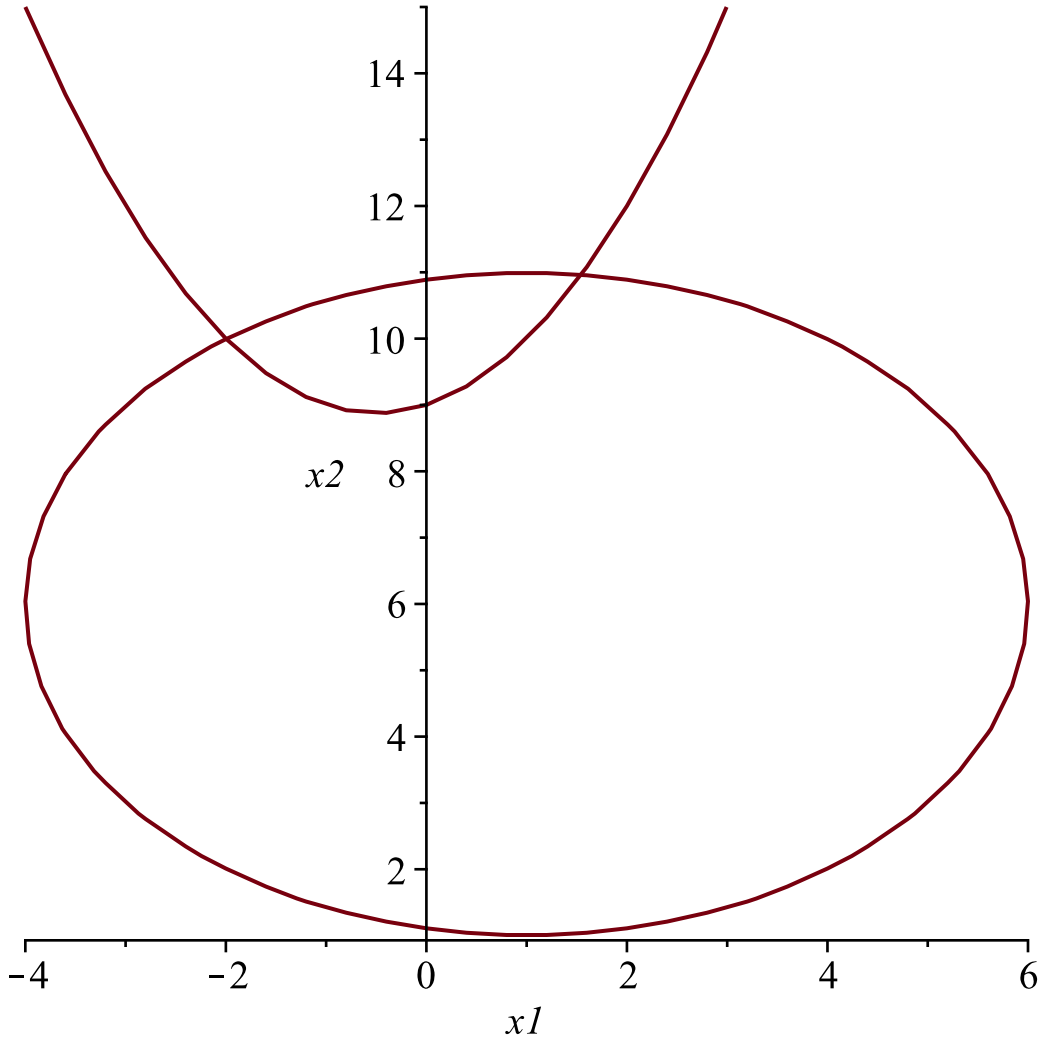
As a comparison

```
> Eigenvalues(A);
```

$$\left[\begin{matrix} 4.10529176051018 + 0.\,I \\ -4.02430097795137 + 0.\,I \\ 0.0286911877140858 + 0.\,I \\ -2.10968197027289 + 0.\,I \end{matrix}\right] \quad \textbf{(16)}$$

Problems 1, 2 on page 648

```
> restart;
  with(plots):
> implicitplot([-x1*(x1+1)+2*x2-18,(x1-1)^2+(x2-6)^2-25],x1=-4..6,
  x2=-1..15);
```




```
> g1:= (x1,x2) -> (2*x2-18)/(x1+1);
```

```
g2:= (x1,x2) -> 6.0+sqrt(25.0-(x1-1)^2);
```

$$g1 := (x1, x2) \rightarrow \frac{2\,x2 - 18}{x1 + 1}$$

$$g2 := (x1, x2) \rightarrow 6.0 + \sqrt{25.0 - (x1 - 1)^2} \quad \textbf{(17)}$$

```
> a:= 2.0; b := 11.0;eps:= 10.0^(-5);
  for j from 1 to 15 do
  x1:=g1(a,b):
  x2:=g2(a,b):
  if max(abs(x1-a),abs(x2-b)) < eps then print(j,max(abs(x1-a),abs
  (x2-b)),x1,x2); fi;
  a:=x1;b:=x2;
  od:
```

$$a := 2.0$$

$$b := 11.0$$

$$eps := 0.00001000000000$$

$$13,\ 0.000008024,\ 1.546944261,\ 10.96999429$$

$$14,\ 0.000003052,\ 1.546947313,\ 10.96999517$$

$$15,\ 0.000001163,\ 1.546946150,\ 10.96999483 \quad \textbf{(18)}$$

```
> a:= -2; b := 10.0;eps:= 10.0^(-5);
  for j from 1 to 5 do
  x1:=g1(a,b):
  x2:=g2(a,b):
  if max(abs(x1-a),abs(x2-b)) < eps then print(j,max(abs(x1-a),abs
  (x2-b)),x1,x2); fi;
  a:=x1;b:=x2;
  od:
```

$$a := -2$$

$$b := 10.0$$

$$eps := 0.00001000000000$$

$$1,\ 0.,\ -2.0,\ 10.00000000$$

$$2,\ 0.,\ -2.000000000,\ 10.00000000$$

$$3,\ 0.,\ -2.000000000,\ 10.00000000$$

$$4,\ 0.,\ -2.000000000,\ 10.00000000$$

$$5,\ 0.,\ -2.000000000,\ 10.00000000 \quad \textbf{(19)}$$

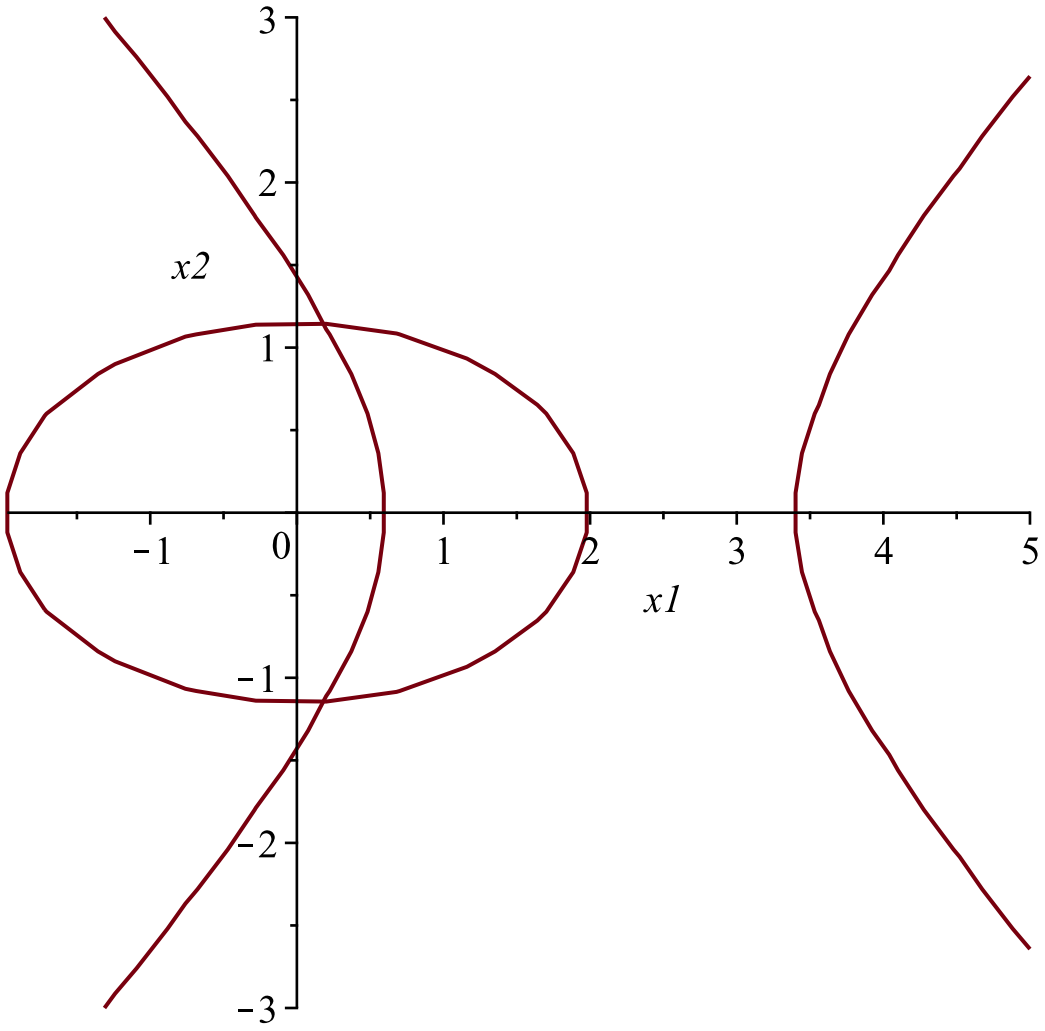
I have cheated here. -2, 10 is the exact solution. Other nearby values DO NOT converge.
(In the end I gave up on trying to find an iteration to give this solution. This illustrates a big flaw of the method, i.e., it relies heavily on luck)

```
> g1:= (x1,x2) -> -0.5+sqrt(2.0*x2-17.75);
  g2:= (x1,x2) -> 6.0+sqrt(25.0-(x1-1)^2);
```

$$g1 := (x1, x2) \rightarrow -0.5 + \sqrt{2.0\,x2 - 17.75}$$

$$g2 := (x1, x2) \rightarrow 6.0 + \sqrt{25.0 - (x1 - 1)^2} \quad \textbf{(20)}$$

```
> restart;
  with(plots):
> implicitplot([x2^2-x1^2+4*x1-2.0,x1^2+3*x2^2-4],x1=-7.0..5,x2=-3.
  .3);
```




```
> g1:= (x1,x2) -> (2.0+x1^2-x2^2)/4.0;
  g2:= (x1,x2) -> sqrt(4.0-x1^2)/sqrt(3.0);
```

$$g1 := (x1, x2) \rightarrow \frac{2.0 + x1^2 - x2^2}{4.0}$$

$$g2 := (x1, x2) \rightarrow \frac{\sqrt{4.0 - x1^2}}{\sqrt{3.0}} \qquad \textbf{(21)}$$

```
> a:= 0.0; b := 1.0;eps:= 10.0^(-5);
  for j from 1 to 10 do
  x1:=g1(a,b):
  x2:=g2(a,b):
  if max(abs(x1-a),abs(x2-b)) < eps then print(j,max(abs(x1-a),abs
  (x2-b)),x1,x2); fi;
  a:=x1;b:=x2;
  od:
```

$$a := 0.$$

$$b := 1.0$$

$$eps := 0.00001000000000$$

$$8,\ 0.0000047056,\ 0.1771255812,\ 1.150163012$$

$$9,\ 9.517\ 10^{-7},\ 0.1771246295,\ 1.150163253$$

$$10,\ 2.233\ 10^{-7},\ 0.1771244062,\ 1.150163302 \tag{22}$$

Problem 1a on page 655

```
> with(LinearAlgebra):
  v:= Vector(2);
  w := Vector(2);
```

$$v := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$w := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{23}$$

```
> Newton2D := proc(f,g,v)
  local J11, J12, J21, J22, A,B, w;
  with(LinearAlgebra);
  J11:=unapply(diff(f(x,y),x),x,y);
  J12:=unapply(diff(f(x,y),y),x,y);
  J21:=unapply(diff(g(x,y),x),x,y);
  J22:=unapply(diff(g(x,y),y),x,y);
  A := (x,y) -> Matrix([[J11(x,y),J12(x,y)],[J21(x,y),J22(x,y)]]);
  w:= Vector([f(v[1],v[2]),g(v[1],v[2])]);
  B:= MatrixInverse(A(v[1],v[2]));
  VectorAdd(v,-MatrixVectorMultiply(B,w));
  end proc:
```

```
> f:= (x1,x2) -> 4*x1^2-20.0*x1+x2^2/4.0+8.0;
  g:= (x1,x2) -> x1*x2^2/2.0 + 2.0*x1 -5.0*x2+8.0;
```

$$f := (x1, x2) \rightarrow 4\,x1^2 - 20.0\,x1 + \frac{x2^2}{4.0} + 8.0$$

$$g := (x1, x2) \rightarrow \frac{x1\,x2^2}{2.0} + 2.0\,x1 - 5.0\,x2 + 8.0 \tag{24}$$

```
> v:= Vector([0.0,0.0]);
  for j from 1 to 4 do
  v:= Newton2D(f,g,v);
  od;
```

$$v := \begin{bmatrix} 0. \\ 0. \end{bmatrix}$$

$$v := \begin{bmatrix} 0.400000000000000 \\ 1.76000000000000 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.495893610552932 \\ 1.98342347419233 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499987614524665 \\ 1.99993704880100 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999999850549 \\ 1.99999999921306 \end{bmatrix} \tag{25}$$

Probl3m 2c on page 655. I should write a program to do Newton for any dimension, but it is messy to do this in Maple!

```
> Newton3D := proc(f,g,h,v)
  local J11, J12, J13, J21, J22, J23, J31, J32, J33, A,B, w;
  with(LinearAlgebra);
  J11:=unapply(diff(f(x,y,z),x),x,y,z);
  J12:=unapply(diff(f(x,y,z),y),x,y,z);
  J13:=unapply(diff(f(x,y,z),z),x,y,z);
  J21:=unapply(diff(g(x,y,z),x),x,y,z);
  J22:=unapply(diff(g(x,y,z),y),x,y,z);
  J23:=unapply(diff(g(x,y,z),z),x,y,z);
  J31:=unapply(diff(h(x,y,z),x),x,y,z);
  J32:=unapply(diff(h(x,y,z),y),x,y,z);
  J33:=unapply(diff(h(x,y,z),z),x,y,z);
  A := (x,y,z) -> Matrix([[J11(x,y,z),J12(x,y,z),J13(x,y,z)],[J21
  (x,y,z),J22(x,y,z),J23(x,y,z)],[J31(x,y,z),J32(x,y,z),J33(x,y,z)]
  ]);
  w:= Vector([f(v[1],v[2],v[3]),g(v[1],v[2],v[3]),h(v[1],v[2],v[3])
  ]);
  B:= MatrixInverse(A(v[1],v[2],v[3]));
  VectorAdd(v,-MatrixVectorMultiply(B,w));
  end proc:
```

```
> f := (x,y,z) -> 15.0*x+y^2-4.0*z-13.0;
  g := (x,y,z) -> x^2+10*y-z-11.0;
  h := (x,y,z) -> y^3-25.0*z+22.0;
```

$$f := (x, y, z) \to 15.0\,x + y^2 - 4.0\,z - 13.0$$

$$g := (x, y, z) \to x^2 + 10\,y - z - 11.0$$

$$h := (x, y, z) \to y^3 - 25.0\,z + 22.0 \qquad \textbf{(26)}$$

```
> v:=Vector(3);
  for j from 1 to 5 do
  v:= Newton3D(f,g,h,v);
  od;
```

$$v := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.10133333333333 \\ 1.18800000000000 \\ 0.880000000000000 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.03668708240559 \\ 1.08592383478135 \\ 0.929779316881078 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.03640047102667 \\ 1.08570655830354 \\ 0.931191437233579 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.03640047032921 \\ 1.08570655074168 \\ 0.931191442315390 \end{bmatrix}$$

$$v := \begin{bmatrix} 1.03640047032921 \\ 1.08570655074168 \\ 0.931191442315390 \end{bmatrix} \quad (27)$$

Problem 9 on page 655

```
> f := (x,y,z) -> 3.0*x -cos(y*z) - 0.5;
  g := (x,y,z) -> x^2-625.0*y^2 - 0.25;
  h := (x,y,z) -> exp(-x*y) + 20.0*z + (10.0*Pi-3.0)/3.0;
```

$$f := (x, y, z) \to 3.0\,x - \cos(y\,z) - 0.5$$

$$g := (x, y, z) \to x^2 - 625.0\,y^2 - 0.25$$

$$h := (x, y, z) \to e^{-xy} + 20.0\,z + \frac{10.0\,\pi - 3.0}{3.0} \quad (28)$$

```
> v:= Vector([1.0,1.0,-1.0]);
```

$$v := \begin{bmatrix} 1.0 \\ 1.0 \\ -1.0 \end{bmatrix} \quad (29)$$

```
> for j from 1 to 10 do
  v:= Newton3D(f,g,h,v);
  od;
```

$$v := \begin{bmatrix} 0.624989254924778 \\ 0.499999982807880 \\ -0.508087671013345 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499520312128955 \\ 0.249974036341644 \\ -0.518190915096175 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499971460902193 \\ 0.124986926187450 \\ -0.520479747183001 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499995533052945 \\ 0.0624934344986921 \\ -0.522037373175438 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999454077803 \\ 0.0312467102606148 \\ -0.522817728137645 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999932577787 \\ 0.0156233534041113 \\ -0.523208207270653 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999991625727 \\ 0.00781167627324747 \\ -0.523403485702105 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999998956635 \\ 0.00390583802977194 \\ -0.523501129945818 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999999869796 \\ 0.00195291898821730 \\ -0.523549952706221 \end{bmatrix}$$

$$v := \begin{bmatrix} 0.499999999983738 \\ 0.000976459487447070 \\ -0.523574364166686 \end{bmatrix} \quad \textbf{(30)}$$

Problem 14
F(x)= Ax-b so JF = A and the Newton iteration reduces to
x --> x - A^{-1}F(x)= x- (x - A^{-1}b) = A^{-1}b, i.e., the solution.

Problem 3a on page 680

```
> restart;
  with(LinearAlgebra):
> Continue := proc(H1,H2,t_Start,v,h)
  local J11, J12, J21, J22, J, H1_t, H2_t, H_t, A,B, w, u;
  with(LinearAlgebra);
  J11:=unapply(diff(H1(x,y,t),x),x,y,t);
  J12:=unapply(diff(H1(x,y,t),y),x,y,t);
  J21:=unapply(diff(H2(x,y,t),x),x,y,t);
  J22:=unapply(diff(H2(x,y,t),y),x,y,t);
  H1_t:=unapply(diff(H1(x,y,t),t),x,y,t);
  H2_t:=unapply(diff(H2(x,y,t),t),x,y,t);
  J := (x,y) -> Matrix([[J11(x,y,t_Start),J12(x,y,t_Start)],[J21(x,
  y,t_Start),J22(x,y,t_Start)]]);
  H_t := (x,y) -> Vector([H1_t(x,y,t_Start),H2_t(x,y,t_Start)]);
  B:= MatrixInverse(J(v[1],v[2]));
  #print(B);
  w:= h*H_t(v[1],v[2]);
  #print(w);
  u:=VectorAdd(v,-MatrixVectorMultiply(B,w));
  end proc;
```

```
Continue := proc(H1, H2, t_Start, v, h)
    local J11, J12, J21, J22, J, H1_t, H2_t, H_t, A, B, w, u;
    with(LinearAlgebra);
    J11 := unapply(diff(H1(x, y, t), x), x, y, t);
    J12 := unapply(diff(H1(x, y, t), y), x, y, t);
    J21 := unapply(diff(H2(x, y, t), x), x, y, t);
    J22 := unapply(diff(H2(x, y, t), y), x, y, t);
    H1_t := unapply(diff(H1(x, y, t), t), x, y, t);
    H2_t := unapply(diff(H2(x, y, t), t), x, y, t);
    J := (x, y) -> Matrix([[J11(x, y, t_Start), J12(x, y, t_Start)], [J21(x, y, t_Start), J22(x, y,
    t_Start)]]);
    H_t := (x, y) -> Vector([H1_t(x, y, t_Start), H2_t(x, y, t_Start)]);
    B := LinearAlgebra:-MatrixInverse(J(v[1], v[2]));
    w := h*H_t(v[1], v[2]);
    u := LinearAlgebra:-VectorAdd(v, -LinearAlgebra:-MatrixVectorMultiply(B, w))
end proc
```

(31)

```
> vv:=Vector([0,0]);
  f1 := (x,y) -> 4.0*x^2 - 20.0*x + 0.25*y^2 + 8.0;
  f2 := (x,y) -> 0.5*x*y^2 + 2.0*x -5.0*y + 8.0;
  H1 := (x,y,t) -> t*f1(x,y)+(1-t)*(f1(x,y)-f1(vv[1],vv[2]));
  H2 := (x,y,t) -> t*f2(x,y)+(1-t)*(f2(x,y)-f2(vv[1],vv[2]));
```

$$vv := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$f1 := (x, y) \to 4.0\,x^2 - 20.0\,x + 0.25\,y^2 + 8.0$$

$$f2 := (x, y) \to 0.5\,x\,y^2 + 2.0\,x - 5.0\,y + 8.0$$

$$H1 := (x, y, t) \to t\,f1(x, y) + (1 - t)\left(f1(x, y) - f1\left(vv_1, vv_2\right)\right)$$

$$H2 := (x, y, t) \to t\,f2(x, y) + (1 - t)\left(f2(x, y) - f2\left(vv_1, vv_2\right)\right)$$

(32)

```
> with(LinearAlgebra):
> N:=2;
  h:= 1.0/N;
  v:= Vector([0.0,0.0]);
  t_Start:=0;
  for j from 1 to N do
  v:=Continue(H1,H2,t_Start,v,h);
  t_Start:=t_Start+h;
  print(t_Start,v);
  od:
```

$$N := 2$$

$$h := 0.5000000000$$

$$v := \begin{bmatrix} 0. \\ 0. \end{bmatrix}$$

$$t\_Start := 0$$

$$\left[\begin{array}{l} 0.5000000000,\ \begin{bmatrix} 0.200000000000000 \\ 0.880000000000000 \end{bmatrix} \\ 1.000000000,\ \begin{bmatrix} 0.440060474808973 \\ 1.82798349201160 \end{bmatrix} \end{array}\right] \tag{33}$$