```
> restart;
> N:=18;
  Digits:=N;
```

$$N := 18$$

$$Digits := 18 \tag{1}$$

```
> x0 := Pi/4.0;
  TrueValue := cos(x0);
  TrueValuePrime := -sin(x0);
```

$$x0 := 0.785398163397448310$$

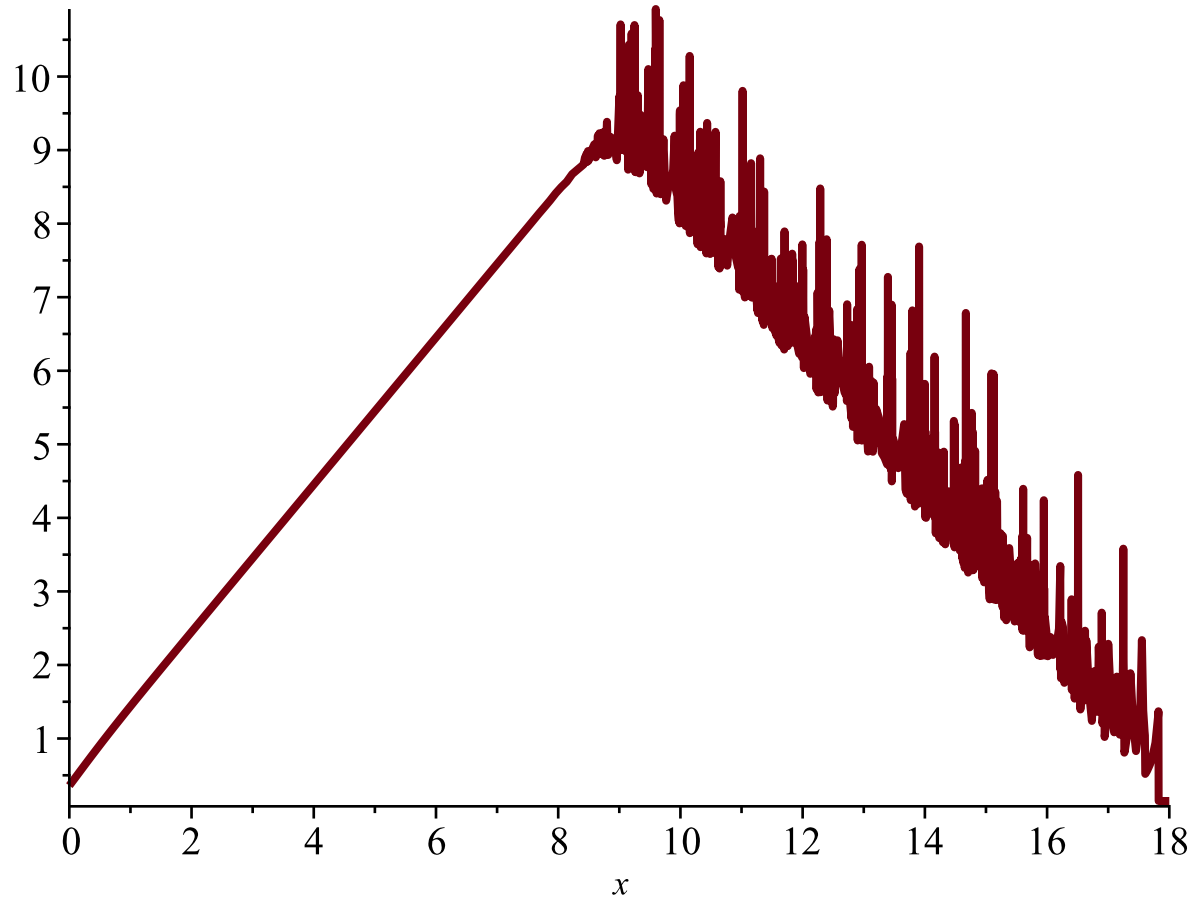
$$TrueValue := 0.707106781186547524$$

$$TrueValuePrime := -0.707106781186547525 \tag{2}$$

```
> g:= x -> (sin(x0+10.0^(-x))-sin(x0))/10.0^(-x);
```

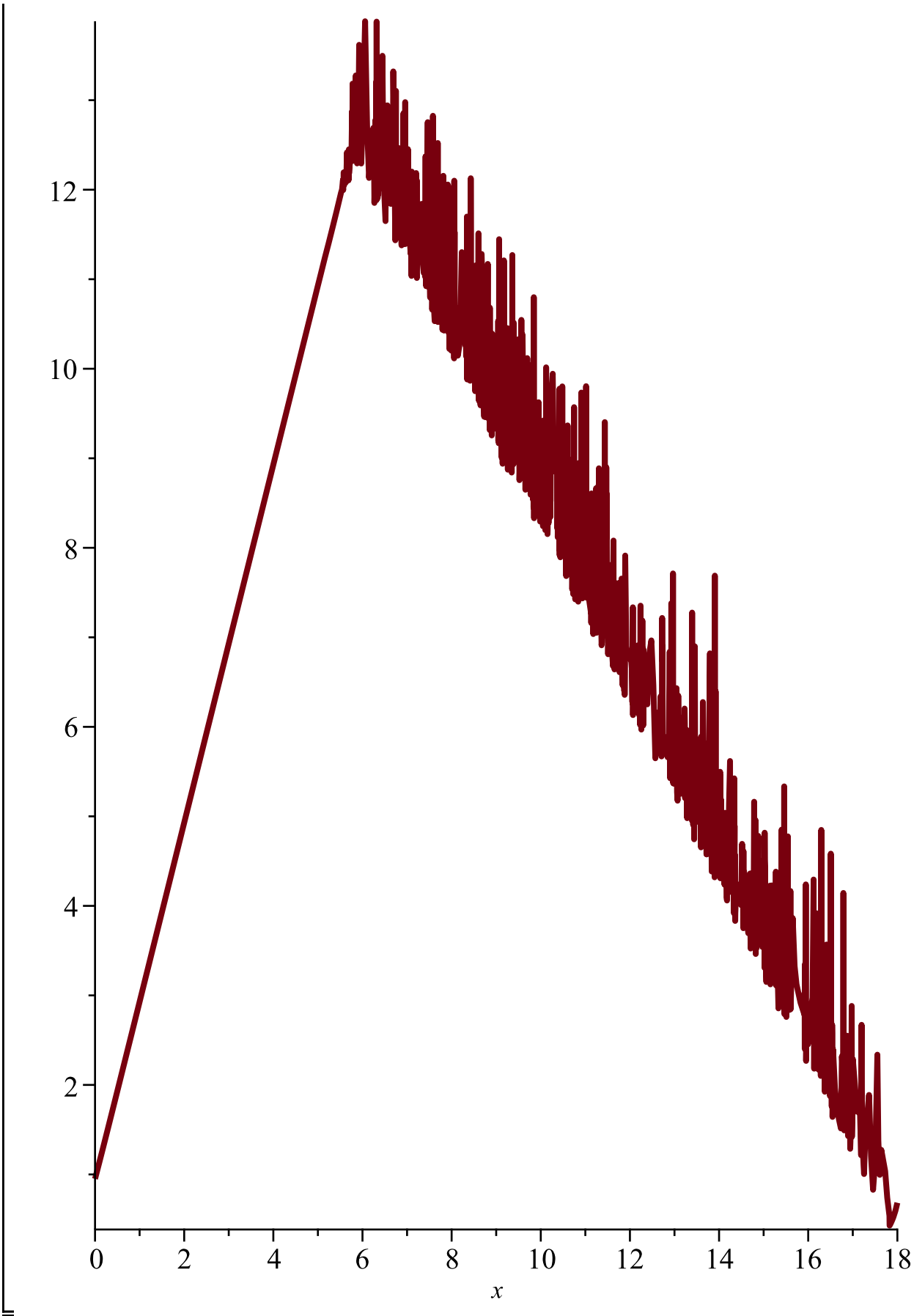
$$g := x \rightarrow \frac{\sin(x0 + 10.0^{-x}) - \sin(x0)}{10.0^{-x}} \tag{3}$$

```
> plot(-log[10](abs(g(x)-TrueValue)),x=0..N,thickness=3,size=[1000,
  350]);
```



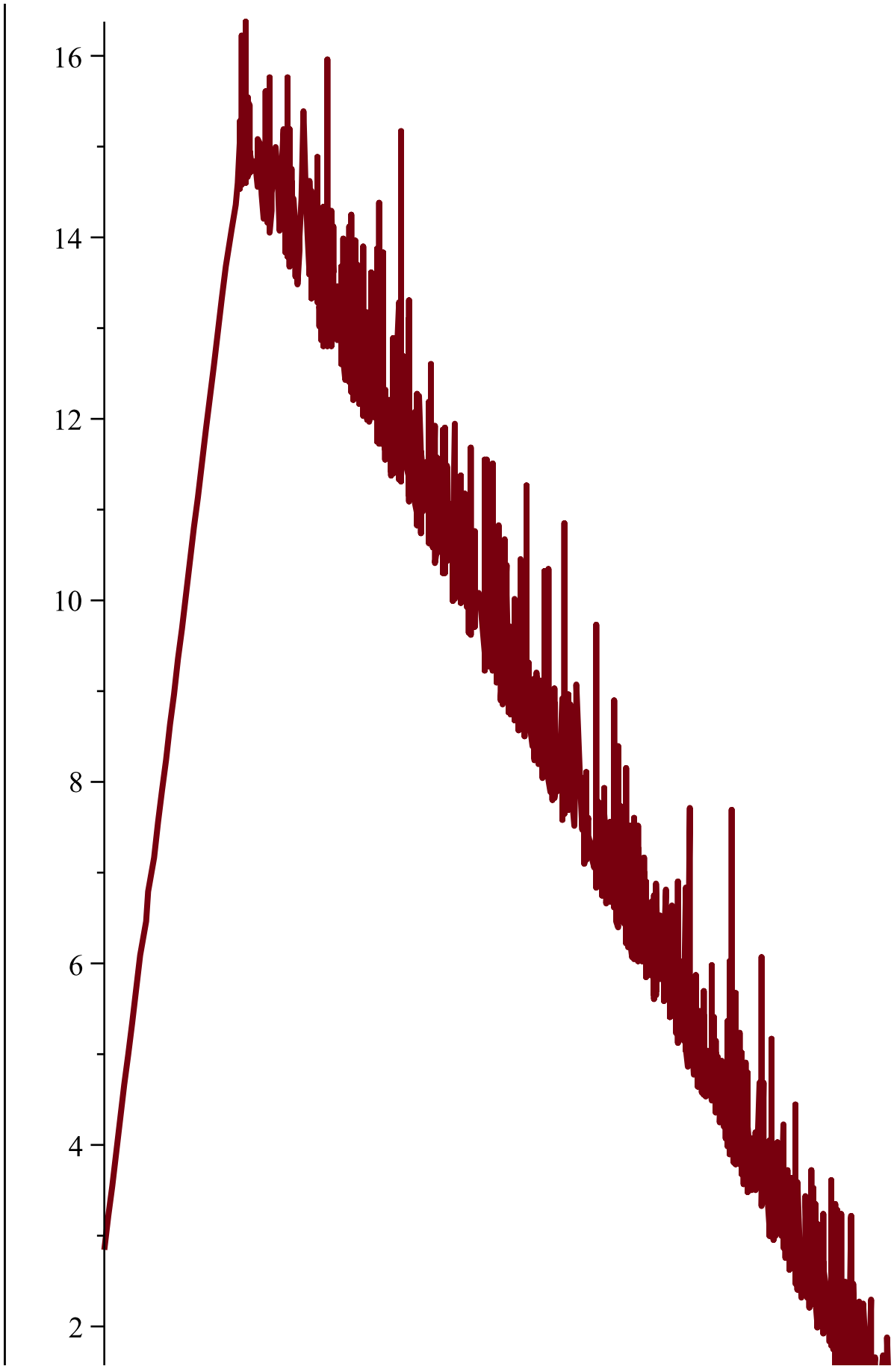

```
> gCent:= x -> (sin(x0+10.0^(-x))-sin(x0-10.0^(-x)))/(2*10.0^(-x));
  plot(-log[10](abs(gCent(x)-TrueValue)),x=0..N,thickness=3,size=
  [1000,670]);
```

$$gCent := x \rightarrow \frac{1}{2} \, \frac{\sin(x0 + 10.0^{-x}) - \sin(x0 - 10.0^{-x})}{10.0^{-x}}$$




```
> plot(-log[10](abs((4*gCent(x+log[10](2.0))-gCent(x))/3.0-
```

```
TrueValue)),x=0..N,thickness=3,size=[1000,800]);
```


16
14
12
10
8
6
4
2

```
> gs:= z -> (sin(x0+z)-sin(x0))/z;
  h:= 10^(-3.0);
```

$$gs := z \to \frac{\sin(x0+z)-\sin(x0)}{z}$$

$$h := 0.00100000000000000000 \tag{4}$$

```
> with(CurveFitting):
  gs(h);
  PolynomialInterpolation([[h,gs(h)],[h/2.0,gs(h/2.0)]],u);
  PolynomialInterpolation([[h,gs(h)],[h/2.0,gs(h/2.0)],[h/4.0,gs
  (h/4.0)]],u);
  PolynomialInterpolation([[h,gs(h)],[h/2.0,gs(h/2.0)],[h/4.0,gs
  (h/4.0)],[h/8.0,gs(h/8.0)]],u);
```

$$0.706753109974292000$$

$$-0.353730115716000000\,u + 0.707106840090008000$$

$$-0.117799546666666667\,u^2 - 0.353553416396000000\,u + 0.707106781190234667$$

$$0.029628952380952380\,u^3 - 0.117851397333333333\,u^2 - 0.353553390470666667\,u + 0.707106781186531048 \tag{5}$$

```
> abs(TrueValue-gs(h));
  abs(TrueValue-subs(u=0,PolynomialInterpolation([[h,gs(h)],[h/2.0,
  gs(h/2.0)]],u)));
  abs(TrueValue-subs(u=0,PolynomialInterpolation([[h,gs(h)],[h/2.0,
  gs(h/2.0)],[h/4.0,gs(h/4.0)]],u)));
  abs(TrueValue-subs(u=0,PolynomialInterpolation([[h,gs(h)],[h/2.0,
  gs(h/2.0)],[h/4.0,gs(h/4.0)],[h/8.0,gs(h/8.0)]],u)));
```

$$0.000353671212255524$$

$$5.8903460476\ 10^{-8}$$

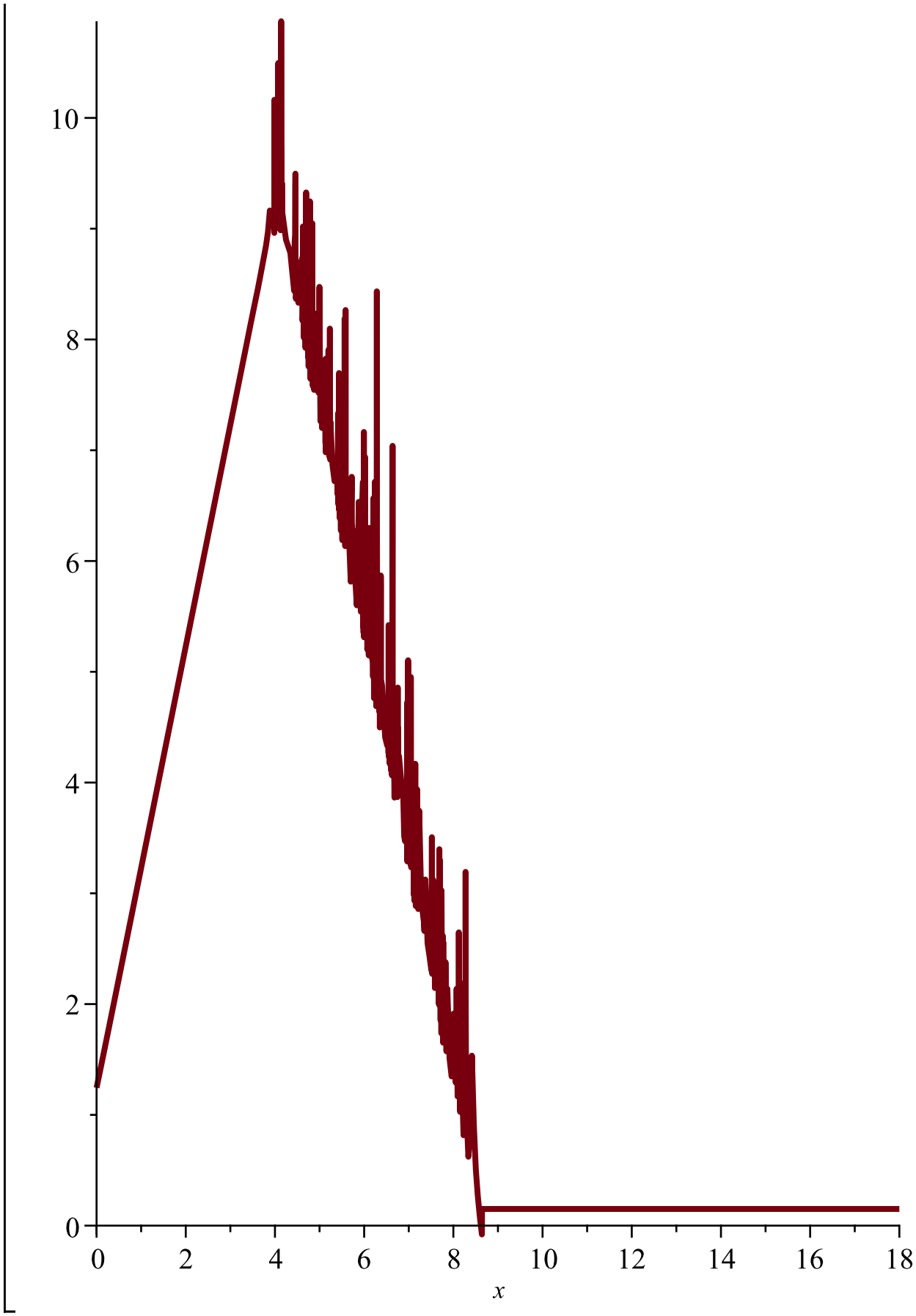
$$3.687143\ 10^{-12}$$

$$1.6476\ 10^{-14} \tag{6}$$

```
> g2 := h -> ((sin(x0+h)+sin(x0-h)) - 2*sin(x0))/h^2;
```

$$g2 := h \to \frac{\sin(x0+h)+\sin(x0-h)-2\sin(x0)}{h^2} \tag{7}$$

```
> plot(-log[10](abs(g2(10.0^(-x))-TrueValuePrime)),x=0..N,
  thickness=3,size=[1000,670]);
```


10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
18
x